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Undergraduate mathematics students' emotional experiences in Linear Algebra courses

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Abstract Little is known about students' emotions in the field of Mathematics Education that go beyond students' emotions in problem solving. To start filling this gap this qualitative research has the aim to identify emotional experiences of undergraduate mathematics students in Linear Algebra courses. In order to obtain data, retrospective focus group interviews were carried out with 27 students. Data analysis is based on the theory of cognitive structure of emotions, which specifies eliciting conditions for each emotion and the variables that affect intensity of emotions. Results show that the participants' emotional experiences in Linear Algebra courses are: satisfaction and disappointment emotions, fear emotions, distress emotions and self-reproach emotions. These emotions are triggered by the appraisal of different situations like the difficulty attributed to Linear Algebra courses, solving problems, asking questions in class, going to the blackboard to solve problems and failure in a course. The students' emotional experiences are based on their appraisal of the situations in terms of specific goals and standards. Some implications for future research on affect in Mathematics Education are discussed.

Keywords Emotions in mathematics education · Students' emotions · Theory of cognitive structure of emotions · Linear Algebra

1 Introduction

1.1 Students' emotions in mathematics education

Most of the research on students' emotions in the field of mathematics education focuses on their role in mathematical problem solving (Adams & McLeod, 1989; De Corte, Depaepe,

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Op't Eynde, & Verschaffel 2011; DeBellis & Goldin, 2006; Goldin, Epstein, Schorr, & Warner, 2011; Goldin, 2000; Mandler, 1989; McCulloch, 2011; Op' T Eynde, De Corte, & Verschaffel, 2006, 2007; Schoenfeld, 1985). Among other results, these studies have confirmed that people tend to experience similar emotions in the process of problem solving (Hannula, 2012).

Goldin (2000), for example, considers that there are frequently recurring sequences of changes in the emotional feelings experienced by individuals engaged in mathematics problem-solving together with their meanings and their cognitive interactions. These sequences are called *affective pathways*; which are sequences of states of feeling that interact with cognitive representations in problem solving. An example of an affective pathway is:

In an (idealized) model, the initial feelings are of curiosity. If the problem has significant depth for the solver, a sense of *puzzlement* will follow, as it proves impossible to satisfy the *curiosity* quickly. Puzzlement does not in itself have unpleasant overtones – but *bewilderment*, the next state in the sequence, may. The latter can include disorientation, a sense of having "lost the thread of the argument" of being "at sea" in the problem... If independent problem solving continues, a lack of perceived progress may result in *frustration*, where the negative affect becomes more powerful and more intrusive. This is associated with the occurrence of an impasse. However, there is still the possibility that a new approach will move the solver back to the sequence of predominately positive affect. *Encouragement* can be followed by *pleasure* as the problem begins to yield, by elation as major insights occur, and by satisfaction with the sense of a problem well solved and with learning that has occurred [emphasis in original] (Goldin, 2000, p. 211).

Similarly Op' T Eynde et al. (2006, 2007) found that students experience different emotions while solving a problem. They can be *annoyed*, *frustrated*, *angry*, *worried*, *anxious*, *relieved*, *happy* or *nervous*. First, for instance, a student can be *worried* during the process of finding a strategy to solve a problem (this is evidenced by students' use of descriptions such as "brow lowering" and "not feeling well"). The student becomes *frustrated* if the solution to the problem does not seem to appear after 10 s ("I don't want to use the calculator", "it does not help me", "but I still want to reach the goal"). Later, *panic* appears and finally *anger* ("come on, what is this all about!"). These results indicate that negative emotions are usually experienced when a student is not able to solve a problem as fluently as expected, and also reveal that, in most cases, the emotional experience triggers students to redirect their behaviour, looking for alternative cognitive strategies to find a solution to the problem.

1.2 Go beyond the problem solving

Little is known about students' emotions in the field of Mathematics Education that go beyond students' emotions in problem solving. Researchers on emotions in mathematics education have pointed out the necessity to move beyond the view of distinguishing emotions in problem solving to focus on emotions during daily activities related to mathematics (Hannula, Pantziara, Wæge, & Schlöglmann, 2010); for example, emotions in mathematics classroom or emotions in mathematics lessons and courses . In addition, Goldin (2014) indicates that the qualitative work focusing on state emotions suggests the "desirability of far more complex descriptions of *affective architecture* in the study of emotion in mathematics education [emphasis added]" (p. 405). Architecture refers to the universal or near-universal functions of emotion, including structures within which emotions occur in human beings: "how emotions are constituted, how they link with cognition, attitudes, beliefs, or values, social interactions, cultural norms and roles, and engagement" (Goldin, 2014, p. 397).

Considering the above, we assumed the task to identify the students' emotions in daily activities in mathematic courses. To achieve this goal we have taken a number of theoretical and methodological choices:

1. We accept that cognition plays an important role in triggering people's emotions (emotion is a reaction supported by a cognitive evaluation called an 'appraisal'). Our acceptance is based on the large body of evidence collected in research on emotions from the point of view of the appraisal theories (Moors, Ellsworth, Scherer, & Frijda, 2013).
2. We use the cognitive structure of emotions theory (Ortony, Clore, & Collins, 1988); which provides a typology of emotional experience in a set of 22 discrete emotions. Some researchers in mathematical education (Di Martino, Coppola, Mollo, Pacelli, & Sabena, 2013; Di Martino & Zan, 2011; Martínez-Sierra & García-González, 2014) have suggested that this theory is appropriate to analyze students' and teachers' emotions. In particular, the study of Martínez-Sierra and García-González (2014) conclude that "most students' emotional experience is related to achievement goals (learn in class, solve a problem, understand the teachers' explanations, interest to learn in class, pass a course, etc.)" (p. 247). Thus, the cognitive structure of emotions theory allows the study of emotions associated with goal oriented mathematics tasks.
3. We accept as valid the verbal self-reports of their emotional experiences —past experienced emotions as narrated by people. This acceptance leads us to offer a way of coding emotional narratives in relation to mathematics. As other researchers in emotions, we believe that emotional phenomenon is multidimensional with cognitive, verbal, motivational, physiological, behavioural and social components. Our position is that research on emotions in Mathematics Education should deal with all possible dimensions of emotional phenomenon; for our particular investigation, we consider the verbal dimension. This is why we consider it necessary to make inquiries from other perspectives and take into account other dimensions of emotional phenomenon as the research using Evan's discursive approach to emotions (Daher, 2015; Evans, Morgan, & Tsatsaroni, 2006).
4. We choose to identify emotional experiences in Linear Algebra courses because previous mathematics education research on Linear Algebra has only focused on epistemology (e.g., Dorier, Robert, Robinet, & Rogalski, 2000; Dorier 1995, 2002a, 2002b) and cognitive aspects. Most of this research is based on APOS (Actions, Processes, Projects, Schema) theory (e.g., Aydin, 2014; Parraguez & Oktaç, 2010) but nothing is known about students' experienced emotions in Linear Algebra. The teaching and learning of Linear Algebra is usually recognised as difficult and the trigger of negative emotions (Dorier, 2002b).

   Students usually feel that they land on another planet, they are overwhelmed by the number of new definitions and the lack of connection with previous knowledge. On the other hand, teachers often feel frustrated and disarmed when faced with the inability of their students to cope with ideas that they consider to be so simple (p. 876).

   Beyond these isolated and almost anecdotal observations little is known about the students' emotions related to Linear Algebra.

1.3 Research question

Considering all the above considerations, the research question of the study reported in this paper is:

RQ. What are students' emotional experiences in Linear Algebra courses?

Our results show some important facts for the research on affect and emotions in Mathematics education:

- It reveals the potential of the cognitive perspective to study emotions. This could help the mathematics education research to move away from the dominant approximation to emotions that gives cognition a limited or non-existent role in emotional experiences.
- It reveals that the identification of the appraisal structure of the emotional experience may refer to other affective dimensions such as motivation to achieve goals.
- It reveals that the emotional experience is not a strictly individual phenomenon but a contextual one.

These results are consistent with the actual trends on affect research in Mathematics education since it highlights that affect and its components form a "dynamical system" (Pepin & Roesken-Winter, 2015).

2 Theoretical framework

2.1 The appraisal theories of emotions

Appraisal theories were proposed and developed to explain how the same event could elicit different emotions in different persons or on different occasions (Moors et al., 2013). There are some basic premises of appraisal theories of emotions and cognitive approaches to emotions: (1) an emotion is a judgment of value —for instance, the importance of a particular event, the pleasure to be with certain person, or the urgency of a specific event. So, an emotion is not just physical like a sneeze, it is an evaluation called an 'appraisal' (Moors et al., 2013; Oatley & Johnson-Laird, 2014); (2) the personal appraisal of a circumstance (current, remembered or imagined) plays a crucial role in the elicitation and differentiation of their emotions (Ellsworth & Scherer, 2009) and (3) emotions are adaptive responses which reflect appraisals of features of the environment that are significant for the person's well-being (Ellsworth, 2013; Moors et al., 2013).

The concept of appraisal is critical because, "in a world that is not fully predictable evaluation of the significance of everyday events and of people with whom one interacts makes emotions central to life" (Oatley & Johnson-Laird, 2014, p. 134). People make judgements about their circumstances by asking different questions such as: is this important to me (concern relevance)?, is something impeding my progress toward a goal?, is something facilitating it (goal conduciveness)?, or has a social norm been broken (compatibility with standards)? Of course, the person does not actually pose such a series of questions each time he or she appraises an event. The different combinations of answers characterize different emotions (Ellsworth & Scherer, 2009).

Appraisal theories of emotion include hypotheses about individual, cultural, and developmental differences (Moors et al., 2013). They account for differences in individual's emotional responses to the same situation. If two people differ in their appraisal of the event's novelty, goal congruence, controllability, or any of the appraisal variables, their emotions will differ. If they have different concerns, one person might appraise the event as furthering those concerns, while other may see it as an obstruction.

Appraisal patterns may also differ in different cultures. For example, Imada and Ellsworth (2011) shows that in similar situations, people from individualist and collectivist cultures

experience different emotions because of the culturally divergent causal attributions for success and failure. Americans reported stronger self-agency emotions (e.g., proud) than did Japanese, whereas Japanese reported a stronger situation-agency emotion (e.g., lucky). So Japanese and Americans differ in their agency appraisals. Japanese are more likely to blame themselves for negative outcomes and to experience shame, while Americans are more likely to blame others and experience anger.

2.2 The appraisal theories and the affective domain in mathematics education

From the previous discussion it can be deduced that appraisal theories of emotions differ from emotions' classical conceptualization of McLeod (1992) in the affective domain. The main difference is the role played by cognition in the emotional process. For appraisal theories, cognition always plays an important role to trigger emotions; but for (McLeod, 1992) "we can think of beliefs, attitudes, and emotions as representing increasing levels of affective involvement, decreasing levels of cognitive involvement, increasing levels of intensity of response, and decreasing levels of response stability" (pp. 578–579). Some other types of affect have been added in recent models; for example, 'values' due to DeBellis and Goldin (2006) and 'mood' due to Evans (2002). Figure 1 shows the relations between these types of affect and cognition (Evans, 2006).

We note then that the dominant vision in Mathematics Education considers that emotions have a very low level of cognitive implication while the other types of affect are more intense and highly unstable. This contrasts with the conceptualization of appraisal theories where cognition is considered as precedent for emotions. Taking this into account and the fact that we focus on asking the participants about their past emotions, we choose to denote this type of emotions as "emotional experiences". These are evoked and remembered experiences that are narrated in the present.

2.3 The theory of the cognitive structure of emotions

For the present study, we choose the cognitive structure of emotions' theory (from now on, OCC theory for the initials of the authors' second name) to identify emotional experiences in students (Ortony et al., 1988). The typology of emotions in this theory specifies a very operative way to analyze the appraisals of the triggering situations (Table 1). We will show this in the Data Analysis Section.

Ortony et al., (1988) propose a cognitive appraisal theory that is structured as a three-branch typology, corresponding to three kinds of stimuli: consequences of events, actions of agents, and aspects of objects. Each kind of stimulus is appraised with respect to one central criterion, called central appraisal variable. An individual judges the following: (1) the *desirability* of an event, i.e., the congruence of its consequences with the individual's goals (an event is pleasant

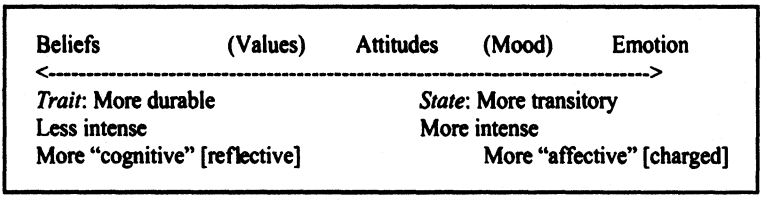

Fig. 1 McLeod's types of affect (Evans, 2006, p. 234)

Table 1 Emotion types according to the OCC theory

Class	Group	Emotions types (sample name)
Reactions to events	Fortunes-of-others	Pleased about an event desirable for someone else (*happy-for*) Pleased about an event undesirable for someone else (*gloating*) Displeased about an event desirable for someone else (*resentment*) Displeased about an event undesirable for someone else (*sorry-for*)
	Prospect-based	Pleased about the prospect of a desirable event (*hope*) Pleased about the confirmation of the prospect of a desirable event (*satisfaction*) Pleased about the disconfirmation of the prospect of an undesirable event (*relief*) Displeased about the disconfirmation of the prospect of a desirable event (*disappointment*) Displeased about the prospect of an undesirable event (*fear*) Displeased about the confirmation of the prospect of an undesirable event (*fears-confirmed*)
	Well-being	Pleased about a desirable event (*joy*) Displeased about an undesirable event (*distress*)
Reactions to agents	Attribution	Approving of one's own praiseworthy action (*pride*) Approving of someone else's praiseworthy action (*appreciation*) Disapproving of one's own blameworthy action (*self-reproach*) Disapproving of someone else's blameworthy action (*reproach*)
Reactions to objects	Attraction	Liking an appealing object (*liking*) Disliking an unappealing object (*disliking*)
Reactions to events and agents simultaneously	Well-being/Attribution	Approving of someone else's praiseworthy action and being pleased about the related desirable event (*gratitude*) Disapproving of someone else's blameworthy action and being displeased about the related undesirable event (*anger*) Approving of one's own praiseworthy action and being pleased about the related desirable event (*gratification*) Disapproving of one's own blameworthy action and being displeased about the related undesirable event (*remorse*)

if it helps the individual to reach his goal, and unpleasant if it prevents him from reaching his goal); (2) the *approbation of an action*, i.e., its conformity to standards; and (3) the *attraction* of an object, i.e., the correspondence of its aspects with the individual's likings.

OCC theory contains twenty-two emotion types that are grouped in three classes and six groups (Table 1). Different types of situations that elicit emotions are classified according to a word or phrase corresponding to a relatively neutral example that fits the type of emotion. For example, to refer to the emotional situation 'pleased about the confirmation of the prospected of a desirable event' OCC theory uses the emotional word *satisfaction* (a *sample name*) because it represents an emotion of relatively neutral valence among all those that express that you are happy about the confirmation of something expected. Similarly, for the emotional

situation 'displeased about the disconfirmation of the prospect of a desirable event', OCC theory uses the emotional word *disappointment* (another *sample name*) because it represents an emotion of relatively neutral valence among other emotional words or phrases that express that you are displeased, such as hopelessness, frustration or heartbreak. The classification of emotions in OCC theory is independent of the words that refer to emotions, as it is a theory about the things that concern denotative words of emotions and not a theory of the words themselves (Ortony et al., 1988).

The theory we propose is decidedly not a theory about emotion words. Indeed, Our characterizations of emotions are intentionally cast in terms that are as independent of emotion words as possible, partly because we believe that the structure of the emotion lexicon is not isomorphic with the structure of emotions themselves, and partly because a theory about emot ions has to be a theory about the kinds of things to which emotion words refer, not about the words themselves (p. 5).

2.3.1 Variables affecting the intensity of emotions

We consider it important to take account of variables that affect the intensity of emotions because their effects can make the difference between having an emotional experience or not. OCC theory specifies global, central and local variables that affect the intensity of the types emotion.

The global variables affect all types of emotions. They include: (1) *sense of reality*, which depends on how much one believes the emotion-inducing situation is real, (2) *proximity*, which depends on how close in psychological space one feels to the situation. The central variables apply to specific types of emotions: (1) *Desirability* applies to event-based emotions, the desirability of an event is appraised in terms of how it facilitates or interferes with the focal goal and the sub-goals that support it. (2) *Blameworthiness* applies to agent-based emotions; *blameworthiness* of an agent's actions is evaluated in terms of a hierarchy of standards. For example, we find someone guilty when this person breaks a rule that we respect.

Local variables are tied to particular groups of emotions. In the case of the analyzed data we only identify local variables of the prospect-based group. These emotions are affected by (1) *likelihood*, which reflects the degree of belief that an anticipated event will occur, (2) *effort*, which reflects the degree to which resources were expended to achieve or avoid an anticipated event.

According to OCC theory, the intensity of any emotion increases according to the degree to which the triggering situations underlying the affective reaction seems real to the person experiencing the emotion (*sense of reality*, global variable) or because of the triggering situations are close together in time (*proximity*, global variable). In all gathered data, these variables are evident because they affect all emotions. This happens because triggering situations are consequence of past and present experiences of the students at the university; experiences that took place very close to the time the interviews were carried out. However, the *central* and *local* variables affect only specific types of emotions depending on student's appraisals. We identify these variables only in some narratives of the students even in the case of the same type of emotion. These are the reasons for the lack of uniformity in the appearance of these variables in the evidence.

2.3.2 Appraisals structures

OCC theory (Ortony et al., 1988, p. 35) conceptualizes three support appraisals structures for changes in the world: (1) *structure of goals* to support appraisals of the desirability of events,

(2) *structure of attitudes* to support appraisals of the appealing of objects and (3) *structure of standards* to support appraisals of the praiseworthiness of actions.

According to OCC theory, we define *goals* as what one wants to achieve. Distinguish three kinds of goals: *Active-pursuit goals* (*AP-goals*), represent the kinds of things one wants to get done in the long term, like passing a course or finishing university. *Interest goals* (*I-goals*), are more routine and short-range goals, these are necessary to achieve A-goals or support them, like passing a test. And *Replenishment goals* (*R-goals*) are those that should be satisfied from time to time in a cyclical nature.

Standards represent the beliefs in terms of which decision assessments are made. We are concerned with *moral or quasi-moral* standards, standards of *behaviour* and standards of *performance. Moral* or *quasi-moral* standards are the guidelines to approve or disapprove the things someone is doing or did. *Behaviour* standards are conventions, norms and other kinds of accepted regularities governing or characterizing social interactions. *Performance* standards are specific role-based norms; we understand them as the roles of being a teacher or student.

3 Methodology

3.1 Context

The research was carried out in a Mathematics Faculty in a northern state of Mexico. This university offers a 4-year mathematical degree organised into eight semesters with five possible specializations: basic mathematics, applied mathematics, mathematical education, statistics and informatics.

This career offers two Linear Algebra courses. Topics in Linear Algebra I include (1) vector spaces, (2) linear transformation, (3) orthogonality, (4) eigenvalues and eigenvectors and (5) matrices calculations. Topics in Linear Algebra II are (1) numerical solutions of systems of linear equations, (2) approximation theory, (3) linear programming, (4) game theory and (5) other applications. These are the first courses that introduce students to the generalizing and unifying theoretical corpus of mathematics (Dorier, 1995, 2002a) and to the obstacle of formalism (Dorier et al., 2000). Thus, students attending Linear Algebra I course are taught some important concepts (e.g., vector spaces) that will be needed at Linear Algebra II course (e.g., linear transformation). So, their performance in Linear Algebra II course depends on their performance in Linear Algebra I course.

Linear algebra lessons are 1 h daily. For 12 years, two female teachers have presented both courses at Mathematics Faculty. The key informant for this research is one of these teachers. She has taught Linear Algebra II since 2001. She suggested this research to the first author of this paper because she wanted to understand the 'suffering' of students in these courses, and also volunteered to help with the logistics of the data gathering.

These courses have a high failure rate: it is always above 50 % but sometimes it can be as high as 80 %. The structure of these Linear Algebra courses is based on teacher explanations. The didactic organisation of the course is highly linked to homework. The teacher also offers a workshop each week for solving problems.

Assessment in both courses of Linear Algebra is the weighted sum of the numerical evaluation of homework and tests. There are several kinds of tests: short tests (consist of two or three questions and are taken once a week), partial tests (taken at the end of every topic) and ordinary tests, these are not real tests but the result of the weighted sum of short tests,

homework and partial tests (80 %) and a final test (20 %) that covers all topics in the course. If a student does not pass the course in this way, they still have the opportunity to take an 'extraordinary test'. This is a single test that evaluates all topics of the course; if a student does not pass the extraordinary test, the option is a 'basis test'. This is the last chance to pass a course without taking the class again.

3.2 Participants

A group of 27 students participated in this research, 14 women and 13 men, aged between 19 and 25 years old. They were students attending courses from second to eighth semester. All of them have already taken at least one of the two courses of Linear Algebra. 13 of them have failed at least one of the two courses. By the time of data gathering: 1) there were 10 students enrolled in the Linear Algebra I course, 9 of them (90 %) participated in the research, 2) there were 12 students enrolled at the course of Linear Algebra II course; 5 of them (41.6 %) participated in the research, and 3) there were 23 students who had already completed Linear Algebra I and II courses; 13 of them participated in the present study (56.5 %).

3.3 Data gathering procedure

We decided to access students' emotions from their reports of experienced emotions because the focus of the research is on the students' subjective experiences of emotions. We are aware that the analysis of narratives of emotional experiences is quite different from the direct analysis of emotions but, like Ortony et al. (1988, p. 8), we are willing "to treat people's reports of their emotions as valid, also because emotions are not themselves linguistic things, but the most readily available non-phenomenal access we have to them is through language".

We decided to use focus group interviews because we observed during previous research at the same university that students feel confident and comfortable in expressing their thoughts, feelings and emotions about various topics in focus group interviews. The focus groups were formed according to the semester the students were attending; because we thought they would have similar academic experiences that would help remind students of triggering situations.

Our key informant asked for volunteer students among those enrolled in the courses of Linear Algebra. All students participated in the focal group interviews but three, who did not have time. The teacher organized the interviews in her office in such a way that students in the same semester were in the same group and no students missed any classes. She was not present during the interviews. There were seven focus groups: three with three students (G2, G3 and G5), two with four students (G6 and G7) and two with five students (G1 and G4). The second author of this paper conducted the focus group interviews during May 2012, 2 months after the semester had started. Despite not having previously met interviewed students, a good rapport was achieved during the interviews; this was noted because they freely expressed their views when they were questioned. The interviewer noted that they were very willing to share their experiences. The questions asked in the focus groups were:

1. How do you generally feel in the Linear Algebra course (I or II)?
2. What kind of situation stresses or distresses you in a Linear Algebra course?
3. How do you feel when you solve a problem in a Linear Algebra course?
4. How do you feel when you cannot solve a problem in a Linear Algebra course?
5. How do you feel the day of a test in a Linear Algebra course?

6. What feelings do you relate to Linear Algebra? Why?
7. If you failed a Linear Algebra course (I or II), how did you feel when you failed? And how did you feel when you finally passed?

Questions 3, 4, 5 and 7 helped us identify key situations that trigger emotional experiences of students: solving problems, the test and the assessment of courses. Our informant proposed these triggering situations. The rest of the questions present an emotional experience for students to remember a triggering situation.

3.4 Data analysis

The students were identified as M*n*-G*k* or F*n*-G*k*. Where M and F indicate the participant's gender, *n* (1 to 4) indicates the participant identification number and *k* (1 to 7) indicates the focus group number. For example F2-G1 indicates a woman in focus group 1 who was the second to speak during the interview.

The videotaped interviews were fully transcribed. For data analysis and presentation of evidence we use the following typographical conventions:

1. ***Concise phrases*** that express all the eliciting conditions of the emotional experiences. In the evidence, we highlight these phrases in ***bold italics***.
2. *Emotion words* that express emotional experience. We highlight emotion words in *italics*.
3. Variables that affect intensity of emotions. We underlined phrases that express intensity in the evidence. We add the name of the intensity variable in curly brackets {e.g., effort} in the phrase where it is identified.
4. We have included explanations in square brackets [e.g., Note] in order to clarify some of the students' expressions.

Due to the daily use of words to express emotions, it may happen that one word refers to different types of emotions. To identify evoked emotions we took into account the eliciting conditions, just as OCC theory suggests. For example, students **F1-G6** and **F2-G6** use different emotion words ("*I feel fine*" and "*I am really happy*") to express their emotional experiences triggered by the successful solving of a problem. Both emotions are *satisfaction* emotions (pleased about the confirmation of the prospect of a desirable event) from the point of view of OCC theory.

F1-G6: I feel really fine when I solve a problem, especially if I did it alone {effort}. It is uplifting.
F2-G6: I am really happy when I solve a problem, because it is so hard {effort}.

The data analysis was conducted following the next steps: (1) The two authors selected an excerpt of students' narrative which contained clearly an emotional experience, (2) the second author did an interpretation of the excerpt identifying types of emotions, appraisals of triggering situations and intensity variables, (3) the first author reviewed the coding made in the step two, (4) if we agree on the coding made in step two we considered it was correct, (5) If we did not agree on the coding made in step two, then the second author went back to analyze the data in the context of the interaction of the focus group and did new interpretations to return to step two.

We intend to keep the sense of the triggering situations and the students' appraisals of these situations in our English translation to present our results. In order to do this, the translation was supported on a professional Mexican translator and proofread by an English native speaker.

4 Results

Table 2 shows the students' emotional experiences in Linear Algebra courses.

We now describe some of the types of emotions that we have identified in the students' narratives focusing on the most important triggering situations.

4.1 Disappointment emotions

In disappointment emotions —displeased about the confirmation of the prospect of a desirable event— we identified the desirable event 'to solve problems'. Students have different meanings for 'problem–solving'. These range from the narrowest sense of exploring a definition by doing concrete exercises (e.g., numerical matrixes calculus) to the widest meaning of demonstrating a theorem. In a general way, we interpret students' meaning of 'problem-solving' as the general tasks that a teacher asked students to perform. Solving problems represent an I-goal that triggers disappointment emotions when not achieved. We present the triggering situation 'solving homework problems' as an example.

Table 2 Students' emotional experiences in Linear Algebra courses

Type of emotion	Triggering situations	Variables that affect intensity	Goals	Standards
Disappointment	Solving problems in class	Effort	I-goal 'solving a problem'	
	Solving homeworks problems			
	Solving problems in a test	Likelihood		
Satisfaction	Solving homeworks problems		I-goal 'solving a problem'	
	Solving problems in a test	Desirability		
Fear	Attributed difficulty of Linear Algebra course		I-goal 'understanding and learning'	
	Asking about doubts in class		I-goal 'understanding and learning'	
	Going to the blackboard to solve problems	Effort and desirability	I-goal 'solving a problem'	
Distress	Attributed difficulty of Linear Algebra course		I-goal 'understanding and learning'	
	Attributed difficulty of homework	Desirability	I-goal 'passing a test'	
	Attributed difficulty of tests		I-goal 'passing a test'	
	Failing the course	Desirability	AP-goal 'passing Linear Algebra course'	
Self-reproach	Delay in studies	Blameworthiness	AP-goal 'passing Linear Algebra course'	Performance standards
	Repeated failure			

4.1.1 Solving homework problems

The difficulty that students attribute to solving problems increases when the problem is a homework assignment. The higher level of difficulty is due to two circumstances: (1) solving problems in class is easier because the teacher explains the procedures, this means that homework is difficult because the teacher cannot help them, and (2) problems in class are easier because they are 'operation exercises' and homework are 'demonstration problems'.

> **M1-G1**: I start [solving a problem] in class and say: –Oh! I got it! But then I realize I don't. ***I spend a lot of time in the same problem,*** *and then I get frustrated,* ***because I could not solve*** *it.* It happens that the teacher gives really simple examples but homework is more complicated.

The desirability to achieve the *I-goal –solving a problem-* affects the intensity of disappointment emotions in such a degree that if the desired event is confirmed the elicited emotion is not disappointment but satisfaction. M2-G6 gives evidence of this situation.

> **M2-G6:** I heave a sigh of relief **when I solve a homework problem** {desirability}. I feel really, really happy, it takes pressure from me.

4.2 Satisfaction emotions

As in the case of disappointment emotions, in satisfaction emotions —pleased about the confirmation of the prospect of a desirable event— the desirable event identified was the I-goal 'solving a problem'. The confirmation of this desirable event triggers satisfaction emotions.

4.2.1 Solving problems in a test

The difficulty attributed to solving problems is even higher in a test because it is an individual activity. The students expressed that their classmates, teacher and books support them in a class, but not in a test. This greater difficulty attribution implies more effort and causes the experienced emotions of satisfaction in a test to be more intense than during class or homework. This shows that students perceive that the I-goal 'solving a problem' in a test is the proof that they really understood the topic.

> **M1-G3:** It is a *great relief, a joy,* ***if I can solve a problem in a test*** {desirability}. I could go home without a worry; I could sleep fine and said: "I can go to sleep at 10:00 ¡yuhu!" [Joy exclamation because they could not sleep well for days in order to study].

4.3 Fear emotions

Students' appraisals of three situations cause them to experience fear emotions —displeased about the prospect of an undesirable event— (a) attributed difficulty of the Linear Algebra course, (b) asking questions in class and (c) going to the blackboard to solve problems. In (a), 'failing the course and not understanding' is the undesirable event that triggers fear. In (b), the undesirable event is the reproach of teacher or classmates for not having the Linear Algebra knowledge. In (c), the undesirable event that triggers fear is not achieving the I-goal 'solving a

problem'. The students expect all these undesirable events because of their experiences in previous courses. We will now present an excerpt of the analysis of situation (c).

4.3.1 Going to the blackboard to solve a problem

Going to the blackboard to solve a problem is a triggering activity for fear emotions caused by the student's lack of security about their performance. In the following testimonies, F2-G3 and F1-G4 talk about their fear of going to the blackboard because they feel insecure while solving problems (I-goal 'solving a problem'). F2-G3, said that this did not happen to her in the Calculus courses. She was comparing her performance in both courses. Furthermore, F1-G4 showed that the desirability of not having to go to the blackboard affects the intensity of her fear emotions in such a degree that she doesn't even want to attend classes. This situation affects her because 'going to the blackboard' was part of her evaluation and it implies an effort that also affected the intensity of her fear emotions.

> **F2-G3:** *I hated going to the blackboard. I thought I was being ridiculous* because I didn't know how to solve the problem. It didn't happen to me in easier courses, like Calculus.
> **F1-G4:** *I was afraid of* ***going to the blackboard****. I got stressed thinking that she would make me go to the blackboard,* and thought, "don't choose me" but I was always the first to be chosen. I even had a nightmare where I didn't know how to solve a problem in the blackboard. This *stresses me* and makes me wish to skip class {desirability}. ***We must get points for participating and I didn't know how to get them*** {effort}.

4.4 Distress emotions

Our analysis identified four triggering situation of distress emotions —displeased about an undesirable event— they are associated with specific undesirable events: (a) attributed difficulty of the Linear Algebra course, (b) attributed difficulty of homework, (c) attributed difficulty of tests and (d) failing the course. In (a), the undesirable event is associated to the I-goal 'understanding and learning'; in (b) and (c), the undesirable event is associated to the I-goal 'passing a test'; in (d) the undesirable event is associated to the AP-goal 'passing Linear Algebra course'. We will present now an excerpt of the analysis of situations (b) and (d).

4.4.1 Attributed difficulty of homework

The difficulty attributed to homework triggers distress emotions. The desirability to achieve the I-goal 'passing a test' intensifies this emotion because students express that doing homework could help them to achieve this goal.

> **F1-G7:** Homework was *stressful* ***because I didn't understand all*** topics. I tried to solve them with other classmates but ***we were in the same condition,*** *I felt I was stuck.* I knew I have to do it because it would help me for the tests {desirability}.

4.4.2 Failing the course

'Passing a course' is an AP-goal for students, they want to finish their degree so they must pass their courses first. Therefore, when students fail a course, their goals are affected and, as a

consequence, they get distressed. The desirability to achieve this goal increases the intensity of these emotions, as can be seen in the testimony of M1-G1:

> **M1-G1:** Failing a course anguishes me because I would get behind in my plans to finish my degree soon {desirability}.

4.5 Self-reproach emotions

Our analysis identified two blameworthy actions that trigger self-reproach emotions — disapproving of one's own blameworthy action— (a) delay in studies and (b) repeated failure. They are both associated with the AP-goal 'passing Linear Algebra course'. Not achieving this goal triggers self-reproach emotions. The difference between distress emotions is that, in self-reproach emotions, the students identify themselves as agents and recognize their own actions as the triggering situations (compare this with (d) in distress emotions). So, negative appraisals of the different possible outcomes for failing a course trigger self-reproach emotions.

4.5.1 Delay in their studies

Students' appraisals of their outcomes depend on certain performance standards; these are based on their role as students. It dictates that students must pass their courses to finish their degrees and not disappoint their parents. So, students' actions are praiseworthy if they follow these rules; in contrast, they are blameworthy. *Blameworthiness* is a variable that affects the intensity of reproach emotions. The emotional words of F1-G7 and M3-G7 denote less intense self-reproach emotions than those used by F3-G5 and F2-G3 (b, next section) because they passed the course after several attempts. So, the two latter have more intense emotions. It is important to point out the case of M3-G7 who feels bad for disappointing his parents.

> **M2-G5:** *I feel frustration* ***because I am supposed to be a student that comes to study and not fail*** {desirability}. It was the first time I failed. I passed two semesters before without studying. **I understood the advisors but I didn't** *follow* **them.** *I react later {blameworthy},* and study as I should; finally, I got a 9 in Linear Algebra.
> **F1-G7:** *I felt bad* because it was the second time I failed the course. *I was disappointed* ***because I was delayed in my degree*** {blameworthy}.
> **M3-G7:** In my case, I felt bad because **my parents were making a strong effort to keep me studying.** I felt I was disappointing them {blameworthy}.

5 Discussion and conclusions

We collected and analysed students' emotional experiences in Linear Algebra courses in a Mexican university. All the narrated experiences were identified through OCC theory. The different emotional experiences were classified according to the students' appraisals of triggering situations. We also identified the variables that intensify emotions, goals and standards that support emotions. This shows that OCC theory, together with our data analysis, is a suitable tool for the analysis of emotional narratives. There are two main reasons for this. First, OCC theory directs our analysis on the identification of students' appraisals of their Linear Algebra courses. So, we step aside from the common words and phrases to refer to

emotional experiences. Second, students only made reference to situations that are significant for them because they were allowed to speak freely. So, the expressed situations are triggering in a strict sense. Their narratives expressed conscious emotional experiences with a high degree of rationalization. This allowed them to communicate the appraisal processes.

5.1 Emotional experiences, goals and standards

The results show that the students' narratives focus on three groups of emotions: prospect-based group (*satisfaction, disappointment and fear*), well-being group (*distress*) and attribution group (*self-reproach*). The main local variables for the intensity of each group are effort and desirability, respectively. Therefore, almost all the emotional experiences of the students are in the class of emotions named reactions to events. Following OCC theory, the students' emotional experience is based on their appraisals of desirable goals that they try to achieve in an active way like solving problems, passing tests or passing the course.

Although OCC theory considers that emotions in the attribution group (*self-reproach*) are triggered by appraisals in terms of social rules, we identified in this study that the triggering situations for these emotions are related to goals. Thus, students' emotions are completely triggered by the achievement of goals. We found three types of goals that trigger all students' emotions: active-pursuit goals, interest goals and replenishment goals.

From the results of our investigation we propose a hypothetical hierarchical structure of appraisal in terms of the students' goals in the Linear Algebra course (Fig. 2). It is hypothetical

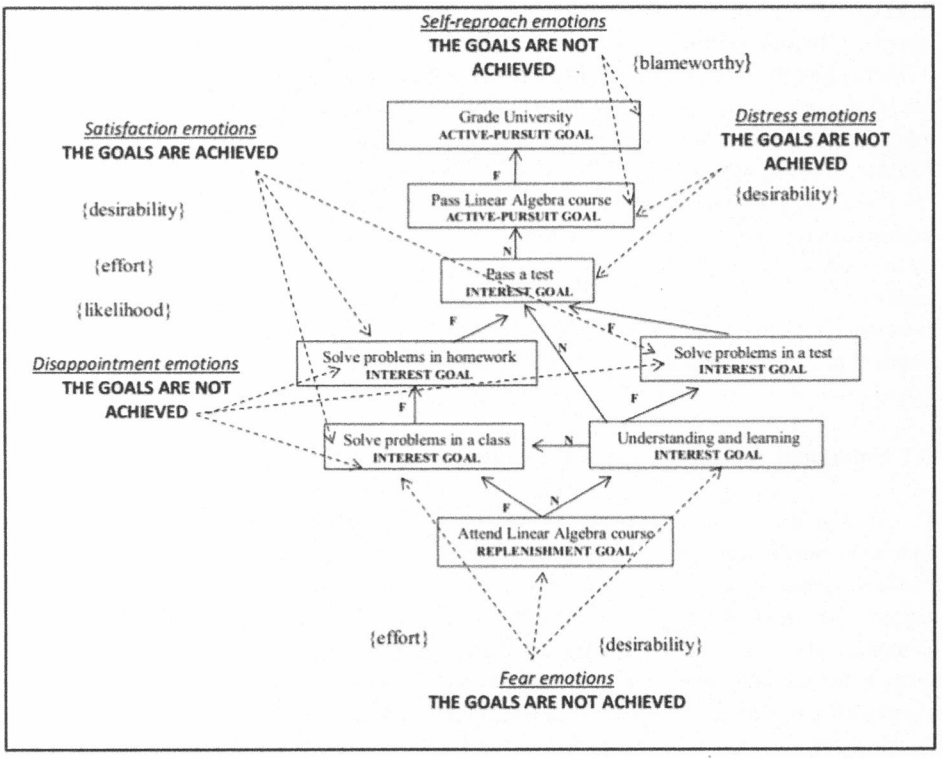

Fig. 2 Students' hypothetic structure of goals

in the sense that it is necessary to ask the participants about the correspondence with their actual goal structure. There are two principal levels in the structure of Fig. 2, the higher level and the lower level. The first level is formed with their expectations and general interests (e.g., AP-goals like 'grade university'); the lower level is formed with more concrete and immediate goals (R-goals like 'attend linear algebra course'). There are some I-goals among these two levels in the structure; these goals help to achieve the highest level from the lowest one. The achievement or not of each goal triggers certain types of emotions and influences the intensity variables shown in brackets. For example, being able to solve problems in a test, in class or homework (I-goals) triggers satisfaction emotions, the variables that affect the intensity in this appraisal are desirability, effort and likelihood. On the contrary, if the goals are not achieved, the disappointment emotions are triggered, the variables mentioned previously also affect these emotions.

Arrows express the different relationships between goals in this structure: an arrow leading from one goal to another represents the possibility that the first goal will affect the achievement of the second one. For example, passing a test is a goal that affects both passing a course and finishing university. The letter N above an arrow denotes that the first goal is necessary to achieve the second one; the letter F denotes that the first goal facilitates the achievement of the second one. For example, passing a test is a necessary condition to pass Linear Algebra Course.

Research in mathematics education has already highlighted the central role of goals in emotional experiences. Hannula (2006) conceptualizes motivation in mathematics as "goals reflected in emotions" because it is possible to direct behaviour through the mechanisms that control emotions. In this regard, some motivational research in mathematics education highlighted "fear of failure" as an important antecedent variable to direct students towards specific achievement goals (Pantziara & Philippou, 2015).

According to some motivational theories in school contexts, the emotions reported in this study are reactions associated with the principle of *competence* in academic motivation: "developing academic competence is both a human need and the expressed goal of schooling" (Turner, Warzon, & Christensen, 2010, p. 3).

These considerations have already been made in more general terms in psychology and educational research. The terms cognition, motivation and emotion have been frequently used as explanatory factors of behaviour and learning. According to Meyer and Turner (2006), the essence of each of these categories can be seen in their function in learning and other behaviour. Cognition deals with information (self and environmental), while motivation directs behaviour. Success or failure in goal-directed behaviour is reflected in emotions. These emotions, in turn, act as a feedback system to cognitive and motivational processes.

5.2 Emotional experience like a contextual phenomenon

Our data indicate that emotional experience is a contextual phenomenon rather than an individual phenomenon. For example, the narratives of the students show that, at the beginning of the semester, there is a strong attribution of a high difficulty to Linear Algebra courses that triggers fear emotions. This is mainly due to the social influence of peers who have already completed the courses and attributed this difficulty. The overall contextual influence can be seen in the fact that all emotions depend on goals set by the curriculum of the course that transcends the individual desires of students (and teachers).

Thus our results indicate, in line with the appraisal theories, that emotions are a sociocultural and contextual phenomenon; understanding by this affirmation that although

emotions are an individual experience, appraisal structures are a social and contextual construction. Thus, in general, from a psychological standpoint, we defend the idea that the human capacity to experience emotions is an innate psychological capacity; but appraisal structures are contextual and social in origin. The foregoing is consistent with the assertions of Radford (2015) who considers that emotions are dynamic processes rather than singular and momentarily subjective emotions; they are socially organized and historically constituted. Hence, like our data show, the goals to support the appraisals of desirability of events are permeated by appraisals of the collective desirability at classroom level.

The previous statements are consistent with other research on emotions in education that states the specificity of emotions in achievement settings. Researchers have defined specific types of emotions and proposed theories based on this. For example, there is an important branch of research on emotions of students that explores *academic* (Pekrun, Goetz, Titz, & Perry, 2002) and *achievement* (Pekrun, Frenzel, Goetz, & Perry, 2007) emotions. Academic emotions are those emotions related to achievement in school. In control-value theory (Pekrun et al., 2007; Pekrun, 2006), achievement emotions are those directly tied to the achievement of activities or outcomes and goals.

5.3 Implications and future research

5.3.1 Implications of using appraisal theories for research on emotions in mathematics education

This research shows that OCC theory is useful to identify students' emotional experiences in mathematics courses. This also indicates the potential of the use of appraisal theories for research on emotions in mathematics education. Other appraisal theories of emotions that we consider relevant in mathematics education are the general appraisal theories and specific theories for educational contexts as the Control-Value Theory of Achievement Emotions (Pekrun et al., 2007; Pekrun, 2006). The task still remains to adapt these emotion theories to the research of emotions in Mathematics Education. In view of our results we consider that Control-Value theory is the most promising for this task; because this theory explains the emotional experience through appraisals in terms of perceived control and value given to a task to achieve goals.

5.3.2 Implications for research on affect in Mathematics Education

The field of affect in Mathematics Education is usually divided into emotions, attitudes, beliefs and values (DeBellis & Goldin, 2006; McLeod, 1992). The results of this research show that emotions can be an 'analytical tool' in the affective domain because it is possible to gain access to other components of the affective domain through the identification of the 'appraisal structure' of the triggering situations of emotions. Our analysis on appraisals based on goals shows this potential with regard to motivation. In methodological terms this means that we can approach other affective variables, like motivation and identity, through the study of emotional experiences. This is consistent with the recent conceptualizations of affect as "dynamic affect systems" (Pepin & Roesken-Winter, 2015) and the general principles of the appraisal theories.

In particular we consider it relevant for future research to study the relationship between emotions and students' and teachers' identities. From the point of view of the appraisal theories we hypothesize that the identity assumed by a person has the function of a structure appraisal that supports their emotional experiences.

5.4 Limitations and future research

5.4.1 About methodology

Trait emotions refer to the students' propensity to experience a particular discrete emotion in the classroom context and are assumed to be a relatively stable attribute of students over time and also across learning situations; *state emotions*, on the other hand, capture the situation-specific emotional state and are thought to vary depending on contextual conditions (Goldin, 2014; Hannula, 2012; Keller, Frenzel, Goetz, Pekrun, & Hensley, 2014). Broadly, the different methodological approaches to the assessment of student emotions allow the identification of trait or state emotions.

We collect data with focus group interviews in this research focus group interviews. These allowed the students to express their emotions widely. The narratives of the students were relatively extensive and rich in meaning because of the interaction between classmates who share everyday experiences in the mathematics classroom. Taken together, we consider that emotions identified in this research are trait emotions because emotional experiences evoked by the students were made in reference to its generic experience in the Linear Algebra courses.

We consider that other methodological approaches –such as individual interviews, diary methods, experience sampling methods or stimulated recall methods– would allow identifying students' state and trait emotions. Diary methods provide frequent reports on the events and experiences of participants' daily lives. In the experience sampling method, participants are asked to stop at certain times and make notes of their experiences in real time. The stimulated recall method is an introspection procedure in which (normally) videotaped passages of behavior are replayed to stimulate recall of the participants' cognitive or affective activity simultaneously. As can be seen, each method is characterized by the proximity of data collection to the moment the 'emotional event' actually occurs; so each method has particular potential to collect students' and teachers' state and trait emotions.

5.4.2 About the questions asked in the focus groups interviews

An inherent limitation of data collection through interviews is the type of questions asked to participants. Some of the questions asked in the focus group interviews have the limitation of imposing some triggering situations on the interviewee (like questions 3 and 7) and request the report of negative emotions (as in questions 2 and 7) without considering asking in an equivalent form for positive emotions. The latter fact may have contributed to obtain that almost all the experiences reported by students were negative.

To overcome these limitations, we suggest that future research explore emotions through self-reports where participants answer more phenomenological questions; i.e., questions directed to the participant's experiences, feelings, beliefs and convictions about the theme in question (like the questions 1 and 6 asked in the focus groups).

5.4.3 About context

This research was conducted in a specific academic context. We consider that it is necessary to keep exploring students' emotions (and teachers') in different academic contexts, different school levels and different mathematical contents. In line with appraisal theories, our hypothesis is that people experience the same emotions but with quite different appraisal structures that are context dependent.

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