Chapter 14

Introduction to an Economic Problem: A Models and Modeling Perspective

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In this chapter, we present an activity for middle school students that was adopted from an economics problem typically used in calculus courses at a university level. It introduced students to a real-life situation, and encouraged them to develop and explain a mathematical model that would help solve a real-life problem. The results demonstrate convincingly that in spite of the common belief, most students can learn useful mathematics in schools. When given the opportunity, and the adequate task and environment, students are able to even go beyond expectancies from their teachers or school standards.

One of the main reasons why a university is a place where knowledge is generated and disseminated is because it provides an enriching environment where ideas can be discussed from a variety of disciplines by subject experts. This chapter is the result of numerous conversations between a professor from the School of Management and School of Mathematics, and a graduate student from the School of Education, who met in an economics course at Purdue University. Both authors converged on the following questions:

1. How can we provide students with rich experiences so that they can develop powerful mathematical ideas?
2. What does it mean to develop powerful mathematical ideas?
3. What does it mean for students to be proficient in their mathematical knowledge?

Coming from different backgrounds, we, the authors, were able to delve into these questions, and enrich each others points of view, and, as a result, provide an introduction to an economic problem from a models and modeling perspective.

THE STARTING POINT

We began our discussions by agreeing upon two main premises taken as a starting point for our conversations. First, we believe it is a myth that only a few students can understand basic mathematic ideas. A quote from Bruner (1960, p. 33) states, "any idea can be represented honestly and usefully in the thought forms of children of school age, and [that] these representations can later be made more powerful and precise the more easily by virtue of this early learning."

The traditional educators "advocate curriculum standards that stress specific, clearly identified mathematical skills, as well as step-by-step procedures for solving problems," (Goldin, in press). These educators also pay careful attention to the answers that students attain and the level of correctness that they demonstrate. Drill and practice methods constitute a huge portion of the time in the classroom to ensure the correct methods in order to achieve the correct answers. Reform educators, on the other hand, advocate curriculum standards in which higher level mathematical reasoning processes are stressed. These include "students finding patterns, making connections, communicating mathematically, and engaging in real-life, contextualized, and open-ended problem solving," (Goldin, p. 5). It is through this open-minded interpretation of education that different ways of students' thinking are verified and encouraged and where a broader variety of students are acknowledged, especially those that are capable, but considered remedial by the traditional standards.

In this chapter, we claim that by being concerned solely with the product of a mathematical procedure, then we are limiting students' thinking to a very narrow view of their capabilities. Thus, we are concerned with the mathematical process of how students attain that product as much as we are interested in their final product. In fact, this can be done from a models and modeling perspective, because when students solve a model-eliciting activity, the process is the product (Doerr and Lesh, chap. 6, this volume).

The second premise is based on a quote attributed to Aristotle: "You've proved that you learned something when you're able to teach it to others." By this, we mean that it is as important for students to develop mathematical ideas, as it is to communicate them to others. This process will not only allow them to converse with others, but will also promote student's development and refinement of these and other ideas. The communication process encourages students to question their own understanding of the ideas and contrast them with others, which in turn, allows them to constantly understand, revise, and refine.

When students enter an undergraduate program where mathematic courses are required, they are usually asked to justify their responses. This is an extremely difficult task for students, who are mostly used to multiple-choice tests, and are used to situations where only correct answers are rewarded. The emphasis in their middle and high school mathematic courses is focused on preparing students to achieve high scores on standardized tests. These tests are usually multiple choice, where further justifications or explanations are not required. Rarely are these types of tests concerned with what students are required to do in their university mathematic courses, and even less to what they have to do when they obtain a job. Thus, there is a large discrepancy between what students are required to do in their middle and high school mathematic courses and what they are expected to do in their university courses and at working settings.

In order to prepare them for future jobs, faculty in universities believe it is important for students to develop their own mathematical ideas from real-life situations, as well as to justify how they obtained their results. In this case, the answer that students give to a problem is as important as the procedure they used to arrive at the solution. This requires from students two important skills: (a) to be able to develop a mathematical model from a real-life situation, and (b) to be able to explain this model to someone else. Neither of these two tasks seem to be a part of most middle and high school mathematic programs. This shows an urgent need for schools to incorporate mathematical activities related to real-life situations that provide students with similar experiences to the ones they encounter in their university mathematic courses as well as in their future jobs.

In this chapter, we present an activity for middle school students that was used with three seventh grade groups. It introduced students to a real-life situation, and encouraged them to develop and explain a mathematical model that would help solve a real-life problem. The activity had to be solved in teams of three students. When students were given the opportunity to explain and justify their thinking to others, it provided teachers with a way to follow students' understanding of the situation. That is, students' mathematical thinking was revealed to the teachers through the explanation that they describe, which provides a powerful evidence of students' mathematical knowledge.

HISTORIC HOTELS: AN ECONOMIC MODEL

One of the basic mathematic ideas that students learn in their high school math courses, and that is recursively utilized in most mathematic courses at an undergraduate and graduate level is quadratic functions. We decided that this powerful mathematic idea could be a good start to our interpretation of students' development of mathematical knowledge.

Based on an economic problem (Aliprantis, 1999) typically used on undergraduate calculus courses that deals with economic concepts, like profit, cost, price, maximization, and equilibrium; and mathematical concepts like recognition of variables, relation between variables (linear and quadratic relations), product of linear relations, and maximization; the two authors designed a model-eliciting activity (Lesh, Hoover, Hole, Kelly, & Post, 2000) that would allow middle school students to develop a mathematical and economic model approaching the concepts mentioned. This activity, Historic Hotels (Appendix A), was intended to encourage students to develop the model from a real-life situation, and incorporated elements so that they could share their model with others.

The real-life context of this activity was given as a newspaper article that described a historic hotel in Indiana, and how it had changed owners through time. The article mentions how it has been difficult for all of them to maintain the historic architecture and ambience of the hotel, in addition to other responsibilities that a good hotel manager is required to do.

The problem statement describes how Mr. Frank Graham, from Elkhart District in Indiana, inherited a historic hotel. He would like to keep it, but is unwilling because of his lack of hotel management experience. The whole community of Elkhart is willing to help him out because this historic hotel represents a major attraction for visitors, and thus, sources of income for everyone in the town. As part of the community, Elkhart Middle School has been assigned to help determine how much should be charged for each of the 80 rooms in the hotel in order to maximize Mr. Graham's profit. From previous experience, they have been told that all rooms are occupied when the daily rate is $60 per room. Each occupied room has a $4 cost for service and maintenance per day. They also have been told that for every dollar increase in the daily $60 rate, there is a vacant room.

Students solving this model-eliciting activity are required to develop a tool for the students in Elkhart Middle School that can help Mr. Graham solve his problem, giving complete instructions on how to use. This tool should be useful even if hotel prices and costs rise, for example, 10 years from now. Students must describe their product through some type of representational media, in order to communicate their tool and its use to the other students and to Mr. Graham.

We implemented a model-eliciting activity in a mid-western public middle school; more precisely with three groups of typical seventh grade students who did not have previous instruction in algebra (one group was remedial). The two teachers had previous experience in implementing model-eliciting activities in their classrooms, but for the students involved in this study, this was their first time.

The implementation consisted of two parts. First, we handed out a copy of the newspaper article, with focusing questions included, the day before the activity took place in the classroom. Therefore, the students were able to read the article as a homework assignment and had a chance to think about the topic that they would be working on the next day in class. The following day, two blocks of time, approximately 50 minutes each, were used for the activity. The first block of time was used for solving the problem and the second for the students to present their solutions to the remainder of the students.

The students worked in teams of three to five to solve the problem, and there were a total of 12 teams. The idea was for the students to work with each other and develop a solution to the problem that has been posed. The teachers were asked to observe the students and not intervene in the students' work time. We did not want the teachers showing students ideal procedures that could assist them in solving the problem since this would defeat the whole purpose of the activity. This request did not pose a problem for the two teachers involved, because they had previous experience implementing these types of activities.

At the beginning of the activity, students were asked to present their solution on an overhead transparency and then show and explain their solution to the class. After the students worked on the problem for about 50 minutes they were requested to give their presentations. At the end of each presentation, the teachers and the remainder of the class had the opportunity to ask questions of the students who were presenting their solutions.

For the purposes of analysis, we collected all of the students' written work. This included all of the paper that the students used while solving the problem and especially their written solutions to the problem and presentations. In addition, one of the researchers wrote field notes as an outside observer and we included this as part of our data.

Once students' work and field notes were collected, we developed different types of coding students' work, according to the representational systems that students used when solving this activity. To do so, one of us looked at each of the team's work individually. After classifying the students' work, we reviewed the students' actual work. We looked for consistency in their reasoning and errors in their mathematics or method. After looking at all of their work, we concluded whether the students found the correct answer or not. Even though this is not the point of the exercise, it was important to note and record. The coding was then discussed, and we both agreed on how the data was going to finally be coded.

The coding was done both, inductively and deductively. We developed a first coding, from Goldin (in press, p. 25). Then, we realized that we did not have evidence for affective representational systems (Goldin, p. 23), because we had not collected videotape for our data. Thus, we went through all of students' work, and developed the following coding:

Type of Representational System

A: Algebraic
C: Chart
G: Graph
L: List
P: Pre-algebraic
T: Text (includes prose with numbers and signs like $).

Based on our data, we also found that students developed different procedures to solve the problem. The method we used to code their procedures is shown here:

Mathematical procedures

1. Multiply the price by the number of occupied rooms. Multiply the maintenance fee by the number of occupied rooms. Subtract the maintenance cost from the total cost of the occupied rooms.

2. Systematical comparison of different numbers of occupied rooms with the corresponding price
   2a: and lowering number of occupied rooms by 5.
   2b: and lowering number of occupied rooms by 1.
3. Attempt to formalize algebraically an iterative process.
4. Unfinished procedure.
5. Introduction of formal terminology (economic/mathematical).

Because in model-eliciting activities the product is the process, we were also interested in how consistent students were in their procedures. That is, if they actually solved the problem the way they described the procedure to Mr. Graham.

Consistency in Procedure
Yes
No
This model-eliciting activity allows for different solutions, depending on how the students interpret the cost for maintenance, so we were also interested in analyzing whether the students wrote an answer (this was not necessary, because the problem asked students to develop a tool and not necessarily to give a numerical answer).

Correct Answer was given
Yes
No
After students' work was coded, we summarized our results in a table format that is shown in the next section for results.

RESULTS

The results of the analysis are summarized in the following chart:

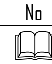
Team	1	2	3	4	5	6	7	8	9	10	11	12
Representation	L T	A L	L T	G P T	T	P T	T	L T	P	A C	T	P T
Mathematical Procedures	1 2a	1 2b 4	1 2b	1 2b 5	1	1 2b	1	1 2a 2b	1	1 2b 5	4	1
Consistency	Yes	Yes	No*	Yes	Yes	Yes	No	Yes	No	No	No	Yes
Answer	No	No	Yes	Yes	No	Yes	Yes	No	N/A	No	No	No
Contextual Influence												

FIG. 14.1. Table of Summarized Results. Historic Hotels Problem.

Relevant Comments From Students' Work

Team 4. *This group was formed by three girls who were in the remedial seventh grade class. They developed their list of operations, and systematically compared the total profit, with different combinations of number of rooms with the corresponding price. They started calculating this list on a separate piece of paper, where they found that the total profit followed a pattern. The values increased, until they got to a certain point (the solution: 68 rooms at $72 per room), and then they started to decrease. When they found this pattern, they immediately asked for a transparency to prepare their presentation. As they started to copy their list to the transparency, they wanted to make sure that their calculations were correctly performed, so they recalculated line by line. As part of their notation, they used dollar signs ($) before the four dollar maintenance fee. As part of their systematization, it was clear that when two of the columns were decreased by one (the number of occupied rooms), the other column increased by one (the price of the room). Unfortunately, they were running out of time, and when they got to line 16, where they were combining 65 occupied rooms at $75, they got confused with their own notation, confusing $4 with 54. This caused them to continue decreasing the values for that column to 53, 52, 51, …Of course, their numerical result for the total profit was affected by this process, and as a result, their hypothesis for the response and patterns found for the maximization of the function was not validated. (Appendix B).

Team 6. Students on this team presented a result for the total profit and added the following note: "This is if they took his taxes out of his pay check." After giving the total profit. This response was very illustrating for us, because it gave us information on how important context is for students when they are solving these types of problems. From this comment we were able to appreciate students' concerns about real-life problems (like taxes), and how they are giving a solution where they are conscious that Mr. Graham will probably need to pay his taxes, and how he should be aware that this method of solving the problem will give him the daily profit, but without calculating the taxes he will have to pay (Appendix C and D).

Team 5. These students introduced formal terminology from the Economics Field. For example: net pay, gross pay, and so forth. (Appendix E).

Team 7. These students clearly and concisely mapped out the tool through prose. They had to develop a tool that would help someone external (Mr. Graham), and they developed a very thorough description of what he was supposed to do in order to solve his problem. (Appendix F).

Team 8. This group used decimal places for the dependent variable (cost of the room) in currency units. (Appendix G).

Representational Systems

In some cases, the representations that the students used were a mixture of one or more of the basic descriptions. We believe this to be an important finding. When students are working with different representations, it is typical that most textbooks or computer activities ask them to give their response in a specific representational system. For example, “give your answer in a table,” “from this chart, develop the graph,” and so forth. What we believe is relevant from students working on this activity is the fact that students were not told or suggested to present their response in a certain form. Nevertheless, they used what they considered most appropriate for the task that they had to develop, and it is interesting to see how: (a) They came with their own representations, that are considered very powerful in the field of mathematics, and (b) about 70% of the students used more than one representational system to express their model, which implies that students were also able to map from one representation to another, and vice versa.

General Solution Patterns

At the beginning of the activity students seem to spend quite some time analyzing what the problem is. At first it is not clear that Mr. Graham would make more profit if not all of the rooms are occupied. Until students are able to overcome this initial assumption, they start operating with different combinations of occupied rooms and its corresponding room rate, and they start to see certain patterns in their response. After students operate on two or three combinations, they find that as the number of occupied rooms decrease, the profit increases. One might even think that it would be easy for students to generalize this as a rule fairly quickly and start decreasing the number of rooms as much as possible. Nevertheless, it is quite clear that when students are working on their solution, they are working with number of occupied rooms, and not only with numbers; simultaneously, they are working with a corresponding room rate, and not just a number with dollar units. Thus, it seems that the context helps the students not to generalize the rule, and continue a slow decrease in the number of rooms. This process, and constant comparison of results for different combinations, allows students to observe interesting patterns in their data, like how the profit increases up to a certain point (maximum), and then starts to decrease. Even though for these students these ideas are not yet formalized in terms of, for example, properties of a quadratic equation, a parabola, or maximization, the fact that these students start realizing these patterns at an early stage in their scholar education should provide them with more powerful tools for when they do encounter these mathematical concepts in a more formalized manner.

Procedures

We noticed throughout the course of our analysis that two main procedures were used by the students to help solve the problem. This was an important distinction for the quest of understanding why students act and think in the ways that they do. The first major mathematical procedure that the students used is taking into account the $4 maintenance fee as part of the original $60 cost of the room. Therefore, when figuring out the maximum profit, they began with $56 and started exploring values from there. On the other hand, other students thought that the original price of the room, \$60, did not include the \$4 maintenance fee. Therefore, these students began with a price of \$64 when figuring out the hotel manager's profit. These two procedures, therefore, elicited two different answers depending on the ways that the students interpreted the problem.

Even though not all of the groups were consistent in their response, or obtained the "correct" answer, the models that they produced were considerably powerful, from the mathematical standpoint; and all of the students excelled, by far, the expectations from the teachers, the standards (NCTM, 2000), and even our initial hypotheses (as researchers).

Bruner states that "any subject can be taught effectively in some intellectually honest form to any child at any stage of development," he also adds that, "no evidence exists to contradict it." The fact that among the three classes there was not a single student that did not understand what the problem was, or that none of the groups were unable to develop a useful response is an important finding, especially if we are considering a task that was based on a typical calculus maximization problem solved by a seventh grade mathematics class. We believe this is considerable evidence that supports Bruner's statement.

CONCLUSIONS

The students developed, in solving the problem, a broad variety of representational systems that helped them express their mathematical ideas through the refinement of their model. Nine of the twelve teams utilized text in their work and most of the students listed their data in one way or another. Roughly half of the teams were consistent in their mathematical procedures. In addition, roughly half of the teams arrived at the correct answer based on the method used, as discussed previously. Although not all of the groups got the correct answer, their work shows very powerful mathematical reasoning. Their ideas about representational systems such as notation, terminology, and so forth are not fully developed, yet show results that were by far beyond expectations from their teachers, or from educational standards (NCTM, 2000). Assessing students' mathematical capabilities by only looking at the correct answer is giving a very limited view of the mathematical ideas that students have developed and what they are capable of accomplishing.

IMPLICATIONS

The implementation of this model-eliciting task was very encouraging from the standpoint of view of researchers, teachers, students, and program evaluators. The role of communication in the modeling activity was an essential part of the task. Working in teams allowed students to develop and refine useful mathematical models and to provide documentation of their learning. The fact that students are not only able to develop their own mathematical and economic ideas, that go beyond expectations from the whole scholar community, but also give evidence of this learning is very promising, especially if we consider that most of these capable students are classified as "below average". This is consistent with our initial premises that all students are capable of learning and developing powerful mathematical concepts, and that communicating them to others promotes the development and refinement of their ideas.

As life changes due, among other things, to technology and globalization, the educational system must also change. Early preparation of students for tasks similar to the ones they will encounter in their future education or job is not only possible (and productive), but should be a requirement.