Chapter 48
Pre-service Mathematics Teachers’ Experiences in Modelling Projects from a Socio-critical Modelling Perspective

Mónica E. Villarreal, Cristina B. Esteley, and Silvina Smith

Abstract Partial findings from a research project aimed at characterising experiences in mathematical modelling (MM) projects carried out by pre-service mathematics teachers are reported. The non-mathematical themes selected and the mathematical content used in designing and developing free MM projects are reported. Aspects of the socio-critical modelling perspective present in such projects are also analysed. This analysis revealed that: (a) the selected themes could be categorized as: socio-economic, ecological, personally relevant, didactical and mathematically focused; (b) the mathematical content involved in the projects were associated with statistics, probability, and analysis. The study of a single project about trash and recyclable collection reveals characteristics of socio critical modelling perspective, difficulties of the MM process, and educational reflections.

48.1 Mathematical Modelling and Teacher Education

The incorporation of applications of mathematical models into real world problems (in the sense of Blum et al. 2003) or the development of modelling activities in mathematics classes at different educational levels is a trend that has spread in recent decades in the international context (Kaiser et al. 2013; Stillman et al. 2013b).
More recently, extracurricular modelling events for students have gained popularity in some places (Stillman et al. 2013a). Although there is agreement that modelling should play an important role in mathematics education, resistance and obstacles related to MM are still present in school and university (Silveira and Caldeira 2012).

In our local context, various (national or state) curriculum documents give some recommendations for working with MM at secondary school level and during mathematics teacher education. In the Curriculum Design for Secondary Education of the state of Córdoba (Ministerio de Educación del Gobierno de la Provincia de Córdoba 2011), it is suggested to consider modelling for solving problems, external and internal to mathematics and to encourage the study of limits of the mathematical model to explain a problem or phenomenon. The curriculum documents also emphasize the relationships between the real world and mathematics through modelling processes; however, most of the activities that are found in textbooks and classrooms are illustrative applications (Muller and Burkhardt 2007) to solve semi-real problems (Skovsmose 2001). Although there are some local experiences at secondary schools working with open MM projects (Villarreal et al. 2010; Villarreal and Esteley 2013), the engagement of teachers, pre-service teachers or students in active modelling is still scarce.

Many mathematics educators believe pre-service teachers should experience MM activities if we intend that modelling become an extended trend at school level (Doerr 2007; Lingefjärd 2007; Widjaja 2013). According to Doerr, “Pre-service teachers need to encounter modelling experiences that provide for a range of contexts and tools and that engage them in meta-level analyses of their modelling activity” (p. 77). Lingefjärd states that “in order for modelling to become a part of a teacher’s functioning and practice, experiences provided for them in the course of their own mathematics learning should assist them in constructing an image of the teaching and learning that is enhanced by modelling” (p. 477). In agreement with these ideas, Widjaja accomplished an exploratory case study with Indonesian pre-service teachers to build awareness of MM, having in mind to provide teachers with knowledge and skills to make possible student-centred lessons.

If we turn our attention to teacher education in Argentina, we can find statements coming from recent official documents, espousing the importance and necessity of introducing MM activities during initial education of mathematics teachers. Meanwhile, the teaching of mathematics at our university for pre-service teachers, usually gives little room for active modelling. It seems that in order to turn around this situation and give account of the curriculum demands, it is necessary to review and act on teacher education.

1 In the Argentinean school system, even though there are national curriculum orientations, the design and implementation of the curriculum are the responsibility of each state.
In agreement with the ideas of Doerr (2007) and Lingefjärd (2007), and considering our local context we decided to propose a MM scenario in which pre-service mathematics teachers have the opportunity of experiencing a complete MM process. A MM scenario is characterised by the presence of a set of spaces, situations, conditions, materials, actions and interactions that give sense to the MM process transforming it into an experience that aims to introduce into educational contexts MM as a pedagogical approach and as a mathematical activity (Esteley 2010).

In our proposal we promoted the creation of MM scenarios characterised by:

(a) the open nature of the activities, due to the free choice of a real world theme to study and posing of questions, and the absence of pre-determined mathematical content to be taught;
(b) the interdisciplinary nature of the work;
(c) the promotion of reflections about mathematics itself, the models created, and the social role of mathematics and MM, and
(d) the domain of the whole modelling process. The creation of such scenarios usually encourages and promotes the treatment of critical social aspects. Modelling activities with these characteristics, can be recognized as belonging to the socio-critical modelling perspective (Kaiser and Sriraman 2006), to which our work belongs. For these authors, such a perspective “emphasises the role of mathematics in society and claims the necessity to support critical thinking about the role of mathematics in society, about the role of and nature of mathematical models and the function of mathematical modelling in society” (p. 306). A main teaching objective for this perspective is the promotion of critical thinking and students’ debates during MM. This particular perspective has roots in Brazil where it is closely related to the ethnomathematics movement. In addition, the socio-critical perspective is strongly related to thematic project work as developed by Skovsmose (1994, 2001) within the framework of Critical Mathematics Education. Literature within this perspective is increasing in the international mathematics education community. Some examples are Araújo (2012), Greer et al. (2007), Julie and Mudaly (2007) and Barbosa (2006) that bring discussions about learners’ or teachers’ engagement with the MM of social issues and how modelling could be used as a tool for critical analysis of various unjust or discriminatory situations or social problematic issues in their contexts. Our work adds to the socio-critical perspective considering the study of MM scenarios for pre-service mathematics teachers, differing from the provision of MM experiences by other teacher educators such as Widjaja (2013) by focusing on development of this perspective in the pre-service teachers.

In summary we decided to create a MM scenario for pre-service teachers and consider such experiences from an investigative point of view with the aims of

(a) analyzing mathematical content that they used to create a mathematical model,
the thematic issues that they selected in designing and developing MM projects, and
the relations with social concerns, and (b) studying characteristics and difficulties of
the socio-critical modelling perspective present in the modelling projects. Regarding this last aim, in this chapter we will restrict the analysis to one particular modelling project due to space limitations.
48.2 Methodological Approach

In order to give account of our aims, we developed a qualitative research, based on data from a larger study on the professional development of pre-service teachers in modelling scenarios. In this chapter, we focus on the analysis of 11 MM projects developed by pre-service teachers from the Faculty of Mathematics, Astronomy and Physics at the University of Córdoba (Argentina). In this institution, the graduate course lasts 4 years with 66% of the syllabus being mathematics courses taught by mathematicians. The remainder of the course deals with educational issues, including mathematics education. During the mathematics courses, the pre-service teachers do not work with authentic MM processes. Mainly, these courses focus on pure mathematics with few applications of ready-made models to extra-mathematical contexts.

The context for our research was within a regular annual course on mathematics education, conducted by two teacher educators, always one of them being a researcher of our team. During this course, the teacher educators and pre-service teachers discussed several trends in mathematics education: problem solving, critical mathematics education, uses of technology in mathematics education, ethnomathematics and mathematical modelling. With the aim of allowing the pre-service teachers to experience a complete process of MM, we asked them to develop, in small groups, MM projects starting from a free choice of a real world theme of interest. At the end, they had to elaborate a written report and they had to present their work to the whole class. The decision for carrying out MM activities in a course of mathematics education was motivated not only by the richness and opportunities that such an environment opens to the future teachers, but also by consideration of the future curriculum demands that they would face as mathematics teachers. This experience was repeated with three cohorts of pre-service teachers, 2010–2012.

Our main data sources were the written reports produced by 11 groups of pre-service mathematics teachers belonging to cohorts 2010–2012, the videotapes of oral presentations produced in 2011 and our field notes.

In our analysis, we first focus on the 11 modelling projects considering the following dimensions: the mathematical content that the pre-service teachers used, the thematic issues, and their relations with social problems. Finally, we study in depth a modelling project developed by one group according to the socio-critical modelling perspective. We also refer to some emergent difficulties and challenges which are relevant for the future professional practice of the pre-service teachers.

48.3 Results and Discussion

In order to present our results, we first offer a general analysis of the 11 modelling projects. Later we focus on a particular project.
48.3.1 The Mathematical Modelling Projects

From a general analysis of the data, specially the written reports, there was diversity in the themes proposed by the pre-service teachers as well as on the issues that seem to motivate the selection of these themes. In order to systematize the data, the projects were first analyzed and then classified according to mathematical content used, thematic issues and their relations with social concerns. Table 48.1 displays the four main mathematical content areas that were evident in the 11 projects considered. As the table shows, the pre-service teachers addressed their problems appealing mostly to statistics, probability or analysis. Within these areas, in general, only a reduced set of tools was used. The projects, in which statistics or probability was used, were based on surveys and frequency tables. Those which appealed to analysis, limited its use almost exclusively to functions from \( \mathbb{R} \) to \( \mathbb{R} \), mostly linear or affine, although some other functions appeared: two variable affine functions in a project about water harvesting in dry areas, exponential and logarithmic functions in a project regarding investment recovery for a given business, and inverse proportionality functions when trash and recyclable collection was studied.

A second analysis allowed us to classify the projects taking into consideration the thematic issues selected by the pre-service teachers. We were able to distribute the projects into five groups, as shown in Table 48.2. These categories were socio-economic issues, ecological issues, personally relevant issues, didactical proposals and mathematically focused.

When we related the themes and problems posed by the pre-service teachers with social issues, we noticed that the projects included in the first two categories had authentic social concerns. We infer this from the nature of the issues posed and the ways the pre-service teachers analyzed these themes in their written or oral presentations. The pre-service teachers who developed the project focused on lack of water, considered such a question as a societal problem and they particularized the study in a rural area of the state of Córdoba which is characterized by serious

<table>
<thead>
<tr>
<th>Mathematical content</th>
<th>Pre-service teachers’ MM projects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics</td>
<td>Household electric energy consumption</td>
</tr>
<tr>
<td></td>
<td>Household water consumption</td>
</tr>
<tr>
<td>Probability</td>
<td>Lottery games</td>
</tr>
<tr>
<td></td>
<td>Genetic transmission and human characteristics</td>
</tr>
<tr>
<td>Linear functions</td>
<td>Waiting time at the university dining hall</td>
</tr>
<tr>
<td></td>
<td>Bottled gas supply in a countryside area</td>
</tr>
<tr>
<td></td>
<td>Soy consumption</td>
</tr>
<tr>
<td></td>
<td>Trash and recyclable collection</td>
</tr>
<tr>
<td>Non-linear functions</td>
<td>Investment recovery for a given business (exp, log)</td>
</tr>
<tr>
<td></td>
<td>Trash and recyclable collection ( f(x) = \frac{c}{x} )</td>
</tr>
<tr>
<td></td>
<td>Water harvesting in dry areas (function of two variables)</td>
</tr>
<tr>
<td>Linear programming</td>
<td>Travel costs for end-of-year school trip</td>
</tr>
</tbody>
</table>
problems of access to water. The constructed model was then critiqued on the basis of the cultural, economical or natural conditions of the selected region. The pre-service teachers who focused their study on issues related to ecological conscience proposed a topic that can be considered open enough as it affects all societies; however, they treated the issues from a local point of view. Whilst two projects particularized the study on water or electrical consumptions in their homes, the third one extended the issue, considering trash and recyclable collection in their city (Córdoba). This distinction conferred a greater relevance to this last project due to the variety of data as well as to the implications of the results in terms of proposals and critiques. We analyze this project in the next sub-section.

We observe that the themes selected highlight different perspectives and relationships between pre-service teachers and the contexts that surround them. Some projects are tightly related to their everyday contexts whilst others broaden their local perspectives and contexts. Some projects focus on micro aspects of society, whilst others on macro aspects. These issues not only become evident in the type of selected environments for the projects but also in the imposed conditions, the data collection and the scope of their conclusions and proposals. These differences derived from the commitment that pre-service teachers are able to establish with their own socio-environmental contexts and with the ways they understand the ideas related to the MM socio-critical perspective as treated during the mathematics education course.

### Table 48.2 Classification of projects according to thematic issues

<table>
<thead>
<tr>
<th>Thematic issues</th>
<th>Pre-service teachers’ MM projects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Socio-economic</td>
<td>The lack of water as a social issue</td>
</tr>
<tr>
<td></td>
<td>Household water consumption</td>
</tr>
<tr>
<td>Ecological conscience</td>
<td>Household electric energy consumption</td>
</tr>
<tr>
<td></td>
<td>Trash and recyclable collection</td>
</tr>
<tr>
<td>A problematic that affects them personally or as a group</td>
<td>Waiting time at the university dining hall</td>
</tr>
<tr>
<td></td>
<td>Bottled gas supply in a countryside area</td>
</tr>
<tr>
<td></td>
<td>Human genes– Interactions</td>
</tr>
<tr>
<td></td>
<td>Soy consumption</td>
</tr>
<tr>
<td>Didactical: a teaching proposal</td>
<td>Travel costs for end-of-year school trip</td>
</tr>
<tr>
<td>Mathematics</td>
<td>Lottery games</td>
</tr>
<tr>
<td></td>
<td>Investment recovery for a given business</td>
</tr>
</tbody>
</table>

#### 48.3.2 The Project About Trash and Recyclable Collection

In this section, we concentrate on one particular case, a project related to trash and recyclable collection. Such a project illustrates the MM process followed by a
group of two pre-service teachers, Irene and Rose. We analyze a piece of their project work. All figures and quotations used belong to the oral presentation that they prepared to show their work to the whole class. This presentation was videotaped.

Irene and Rose referred to the difficulties they had in selecting a theme and posing problems to solve. They mentioned all the themes that came to mind as a kind of “brain storm”: petroleum reserves and types of uses, car accidents, afforestation and deforestation, transportation, music and mathematics and trash and recyclable collection, the last being the selected theme. The reasons for their final choice were twofold: they recognized that recyclable collection has a social function and that it was a local problematic of our city, Córdoba. After selecting the theme, they posed several questions: “How much recyclable trash is collected per day/week? Is it classified? What are the classes? How many people have this service? Does everyone classify the trash?” They then looked for information to answer their questions. Separate trash collection and the environmental and social aspects related to it, including the benefits of recycling were investigated. They interviewed an employee of the city government in charge of the city recycling program and visited the website for this program. Finally, the results of the 2010 census of population and housing in Córdoba were obtained.

Considering the questions they had raised, Irene and Rose were concerned with the mathematical complexity that would be involved in their models. During their oral presentation, they said that at the beginning of their project they wanted to obtain a sophisticated formula, or to apply more complicated mathematics, not just linear models. Then, after a very thorough search of the internet and discussion of many issues concerning trash, they decided that the main aim of their project would be “Modelling to raise awareness” and not modelling “to obtain a super formula”. This decision is evidence of the pre-service teachers’ concerns with social issues related to the trash. In this case, mathematics is subordinated to a social aim, it is a tool to understand the phenomenon and think about it.

Considering all the data and information they collected, the pair raised the following hypotheses:

• The inhabitants of Córdoba aren’t aware of the amount of recyclable trash they produce.
• The trash that one person produces, each year, is more than his own weight.
• A large percentage of trash that could be recycled is thrown away with regular trash.

After establishing their first hypotheses, they considered the following data: the city of Córdoba had 1,329,604 inhabitants (2010 census); the inhabitants of Córdoba produce 1,500,000 kg of trash per day; 31 % of the population sort the trash; 28 t of trash are collected per day. Based on these data, Irene and Rose raised an additional hypothesis: Suppose that every person produces a similar amount of recyclable trash. With this assumption and using all the available information, Irene and Rose started to produce their linear models in the form of numerical indicators, such as that shown in Fig. 48.1a, considering the quotient between the
total amount of trash collected per day and the total population of the city of Córdoba. In this way, they obtained as a first numerical indicator: the inhabitants of Córdoba produce 1.3 kg of trash per day per person.

Figure 48.1b shows another indicator: if 31% of the population sorts the trash and that means that we have 28 t of recyclable trash per day, then, supposing that every person produces the same amount of recyclable trash per day, the inhabitants of Córdoba produce 90.32 t of recyclable trash per day. After this calculation, Irene and Rose raised the following question: “Do you have any notion of how much is 90.32 t?” In order to answer this question, they made a video. The video presents a visual representation intended to give a notion of what percentage of the recyclable trash is being recycled in Córdoba (see Fig. 48.2).

The video starts by proposing the viewer thinks of a 90 kg man. Under this assumption, it would take 1,003 men to make 90.32 t. Then, a sequence of images of “trash men” (men wearing trash) begins to appear up to 1,003 units. After that, the following texts appear: “Only 31% of those 90.32 t goes to recycle collection. The rest goes to regular refuse collection”. Finally, the image of 1,003 trash men is
divided representing the percentage of recycled men with the final text stating: “That means that only 311 ‘trash men’ are recovered and recycled. The remaining 692 men are thrown away as waste”. This video is a way of giving account of Irene and Rose’s aim: modelling to raise awareness.

Irene and Rose concluded their oral presentation by saying that the theme they selected had many possibilities to be treated at the school level. They also previewed ways to continue studying the problematic of trash having in mind a new aim: “Modelling to encourage waste recycling”.

During the analysis of Irene and Rose’s work, we focused on two main aspects: (1) difficulties that they faced during the modelling projects and, (2) educational reflections that they made when they were immersed in a modelling project. Considering the difficulties, we could identify two. Firstly: the difficulty in selection of theme was evident in the way they presented this issue as a “Brain storm”. The criteria for selection were related to social concerns. In their talk, they asserted: “We will concentrate on a local problematic with a social function”. A second difficulty is that the pre-service teachers’ strong mathematical background and the institutional context imposed certain restrictions regarding what mathematics to use. Irene said: “We thought that a model should be a complicated formula involving many variables. It took us some time to understand that the phenomenon could be described linearly…” From these words, we can infer that they felt that the kind of mathematical content they were using was not at the level expected for them by their mathematics teachers at the university. Regarding the second aspect, we will use their own words to note educational reflections that they made while they were immersed in a modelling project.

Rose: We were constantly moving from the role of students to the role of teacher. As students, we were doing a project but, at the same time, we wondered what may happen if we propose such a kind of project for a student (oral presentation).

Irene: I also thought that this could be perfectly done with children in a much more entertaining way than doing… those typical exercises of the paradigm of exercise that don’t lead anywhere… I was interested in the project because you can see the mathematics of reality, you can interpret the mathematics in everyday reality… this is a critical reading of everyday mathematics… of what we do everyday… and there, you have the percentages (oral presentation).

The words of Irene and Rose illustrate a meta-level analysis of their modelling activity focusing on their future activity as teachers. Their experience with modelling seems to provide a view on “mathematics of reality” and its relation with a particular mathematical content (percentages). The reflection about their own mathematics let them imagine possible and future learning contexts related to everyday mathematics.
48.4 Concluding Remarks

Considering the 11 modelling projects produced by the pre-service teachers, the results have provided evidence and examples of the peculiarities of working in MM scenarios in which the pre-service teacher has the opportunity to freely select themes and pose problems. The different thematic issues considered for the MM projects offer to future teachers conditions to think about the use of mathematics in different real contexts, imagine MM scenarios for their future classes, and discuss the role of mathematics to treat social concerns.

Considering the work done by Irene and Rose, we would like to remark that, as other pre-service teachers in our study, they were able to build up a MM project developed from a socio-critical modelling perspective. Such a project promoted reflections about mathematics itself, the models created, and the social role of mathematics. In order to build up the project, Irene and Rose had to overcome two main obstacles: one of them was related to the selection of significant themes and problems and the other one was associated with their experience as students in the institutional context of the university. These obstacles were overcome when they realized that they were having a new learning experience. The new experience had to do with: (1) a new way of doing mathematics and understanding its implications for the real world; and (2) the possibility of imagining themselves as mathematics teachers.

Our research suggests that when pre-service teachers develop free MM projects in the context of a regular mathematics education course, they are able to discuss educational issues related to MM. Our MM scenario differs from other proposed for university students, such as the one studied by Caron and Bélair (2007). In their case, open-ended MM projects were carried out by the students as an assignment for a MM course in which they worked with real world themes selected by the instructor, and the focus was on the development of modelling competences among the students.

In line with the ideas of Doerr (2007) and Lingefjärd (2007), we consider that our findings bring evidence of the importance of MM experiences for future teachers, such as the work of Widjaja (2013) also brings (although in that case the pre-service teachers worked with a single assigned modelling task). The experiences we report also contribute to the discussion of social issues related to mathematics, which we consider is an important, but sometimes forgotten, dimension of mathematical education.

In summary, we can assert that experiences with MM during pre-service teacher education contribute to the final educational aim of promoting and extending MM as an important pedagogical trend and mathematical activity in school. In this sense, although it is not the focus of this chapter, we would like to emphasize that the MM experiences lived by our pre-service teachers encouraged some of them to create didactical proposals with a different kind of open modelling tasks to carry out during their supervised period of teaching practice at schools.


