Chapter 19
Mathematical Modelling and Culture: An Empirical Study

Jhony Alexander Villa-Ochoa and Mario J. Berrío

Abstract This chapter presents partial results of a qualitative case study involving five students from a rural educational institution. The students, motivated by making sense of mathematics beyond the classroom, chose topics for the mathematical modelling of coffee farming. They detected some variables found in the cultural practice of coffee growing and then limited the modelling problem so some of their beliefs began to emerge. Results show that some students' beliefs about culture or contextual knowledge did not remain static throughout the study.

19.1 Introduction

In Colombia, the *Ministerio de Educación Nacional* (Ministry of National Education-MEN) has stated that mathematics teaching in primary and secondary school must consider global and local issues in line with education for all; likewise it must consider the diversity, inter-culturalism and the education of citizens with enough responsibility to enjoy their democratic rights and fulfill their duties (Ministerio de Educación Nacional-Colombia 2006). In this sense, it is reasonable to suggest that school mathematics should recognize the importance of situations, problems and phenomena that emerge from social and cultural contexts thus considering these elements as supporting the constitution of mathematical school knowledge. Thus, mathematical modelling, when involved in the study of such contexts through mathematics, seems to be assumed as an activity that is coherent with the goals and ideals suggested for Colombian mathematical curricula.

To us mathematical modelling is a process of studying a phenomenon or situation through mathematics. In this sense, "Modeling can be understood as a pedagogical approach that emphasizes students' choice of a problem to be investigated in the classroom" (Borba and Villarreal 2005, p. 29). Thereby, mathematical

J.A. Villa-Ochoa (✉) • M.J. Berrío
Facultad de Educación, Universidad de Antioquia; RECOMEM,
Bloque 9-415, Medellín 050010, Colombia
e-mail: jhony.villa@udea.edu.co; marioberrio7@hotmail.com

G.A. Stillman et al. (eds.), *Mathematical Modelling in Education Research and Practice*, International Perspectives on the Teaching and Learning of Mathematical Modelling, DOI 10.1007/978-3-319-18272-8_19

modelling not only defines its scope and some philosophical discussions (mainly those related to the transition from the real world, extra-mathematical considerations, and the nature of knowledge of the nature of production practices), but also gives such phenomena and those situations a constitutive role in the modelling activity. According to Borba and Villarreal (2005), students play an active role in modelling, "instead of being just the recipients of tasks designed by others" (p. 29). These considerations make *project work* a favourable setting for mathematical modelling. In this setting, the topics and problems students deal with emerge as interactions between them, the context and their teacher. This establishes mathematical modelling as an unstructured activity different from contextualised tasks which generally are given in text books. Hence in our study, modelling is not restricted to the task of resolving contextualized tasks, even when these tasks include authentic contexts provided by a teacher or any other means different from students' everyday life and their culture.

This chapter reports part of a much larger study that identified some elements that take place in the (re)construction that students make of mathematical models when they are immersed in a specific cultural context. It analyses some aspects of student performance in mathematical modelling in the culture of coffee farmers. The focus research question was: What interactions occur between knowledge of coffee farming and school mathematics through mathematical modelling?

19.2 Roles of Mathematical Modelling in Culture

In international research, some relationships between mathematical modelling, society and culture have been recognized. In particular, Christiansen (1999) refers to Niss's work (1990) to highlight the importance of individuals being able to reflect critically on models and their applications, as well as, to recognize that mathematics plays an important role in shaping the limits of our activities. Likewise, Christiansen emphasizes the fact that mathematics is implicit in culture and society. In this sense, both Blomhøj (2004) and Blum and Borromeo Ferri (2009) have stated the importance of modelling as a process that involves mathematics learning founded in students' reality; also, through this process, other views of mathematics can be generated; but beyond that, other views can be developed about the contexts themselves. In particular, mathematical modelling makes connections between students' daily life experiences and mathematics, which, besides motivating students, place mathematics in culture as a means for describing and understanding daily life activities (Blomhøj 2004).

The role of mathematical modelling has gone beyond an emphasis on cognitive and conceptual processes of mathematics; this is done in order to lead to aspects linking mathematics with critical societal issues and culture. Both Barbosa (2006) and Araújo (2009) have globally contributed to exhibiting the main aspects of a socio-critical perspective on modelling. Araújo (2009) and Rosa et al. (2012) have emphasized the role of mathematical modelling, not only to explain situations that emerge from reality, but also as a way to enable students to have a critical position facing social demands, as well as modifying, and transforming the world.

19.3 The Study

Since this study was focused on the elements that students take into account in re-constructing mathematical models based on their own cultural situations, it was necessary to address the research from a qualitative perspective using case studies. According to Stake (2007), this kind of study allows investigation of the particularity and complexity of a single case to understand its activity within important circumstances. This study, in its first stage, involved a group of 29 students from a rural school located in a town whose economy is based on coffee growing. These students took a field trip around areas in the vicinity of their school with the purpose of establishing a dependence relationship between variables. Students gathered in teams. Each team chose its *work project* topic. This chapter analyses the oral and written productions of one team chosen because they could see mathematics involved in the context of coffee growing, which is a part of their everyday life and of their families' lives. The topic that emerged in these discussions was related to the number of trees that could be planted on a plot of land according to topographical features (flat or sloping) typical of this mountainous region.

Other stages the students conducted in this research were related to:

- a review of the mathematical models used by the *Federación Colombiana de Caficultores* (Colombian Federation of Coffee Growers),
- a (re)construction of a new mathematical model (e.g., models with trigonometric functions),
- an approval of a proposed mathematical model,
- a mathematical model tracking what emerges from a triangular planting method, and
- contrasting this research information with an expert.

Questionnaires were used and there were student-researcher (teacher) discussions. The researcher was able to ask questions allowing him to probe in detail students' considerations. Data were collected through logbooks (drawn up by a researcher), and documents (drawn up by students). Student discussions were videotaped. Subsequently, we digitized the documents, and watched the videos to determine which episodes could be analysed. While the data were collected, the information was compared in parallel, so a general idea of the content could emerge (Creswell 2008). Similarly, an emerging categorization process was developed, so that each piece of evidence coming from each source of information was assembled with the other pieces and a triangulation process was carried out.

19.4 Some Findings

This chapter is based on the results obtained in the early stages of the research. In these stages, it was possible to observe that mathematical modelling, not only has implications for learning mathematics, but also it involves relationships with students' culture.

Students, after being involved in their chosen contexts and in discussions regarding contextual mathematics, took on a topic related to the number of trees that could be planted on a slope compared to the number of trees that could be planted on flat land. Once the situation was defined, the students were divided into two teams. Ana and Felipe formed Team 1; both of them were from Year 11, and team 2 included Maria (Year 10), and Karla and Jaime (Year 11). The names used in this chapter are pseudonyms. In the first part of the process, the students were committed to the analysis of a rectangular sloping field; the dimensions of the plot can be seen in Fig. 19.1a. Encouraged by their teacher, students first divided the figure into squares with 100 m sides in order to have one hectare (10,000 m^2, agricultural unit of measurement). Figure 19.1b, c shows this process.

In Fig. 19.1b, c one can see that both teams sketched the hectare on a slope with the dimensions of the land. Thus, they showed that they considered this measure as *a square that suits any land regardless of its slope*. Our knowledge of the region allows us to consider that this is an idea that seems to be widely present among people in the agricultural sector. This idea of land area is related to terrain dimensions, but not to its slope. The idea seems to emerge from the contact that people have with their context.

To obtain more evidence about the characteristics that the students assigned to the notion of area –which we will call "Euclidean surface" – the teacher invited them to think about the following situation: "If you were to buy a rectangular piece of land and you had two options, A or B, which one would you buy and why? You can consider a plot of land to grow coffee and suppose that every square (i.e., a hectare) costs the same". See Fig. 19.2.

Facing this situation, students gathered, discussed, and then gave their opinions. For Team 1, Ana reported her team's work and stated:

> Plot A is 13,200 m^2 and plot B is only one hectare. [We would buy land A] because land A is wider, and despite having a slope, it can be used [for] the "tresbolillos" method [a triangular planting method] so there would be [many] more trees in a plot of land.

For Team 2, Maria reported her team's work and stated:

> We would buy land A, because we calculated the area. The area would be 13,200 m^2 [Land A], while land B would be about 10,000 [square] metres, then more trees would fit into this [Land A].

Both team 1 and 2 agreed that the land that should be bought was land A, because it was 13,200 m^2 (sloping), while land B had only 10,000 m^2 (horizontal). On the other hand, team 2 mentioned the direct relationship between the area and the number of trees: "the larger the area, the greater the number of trees that fit". It is

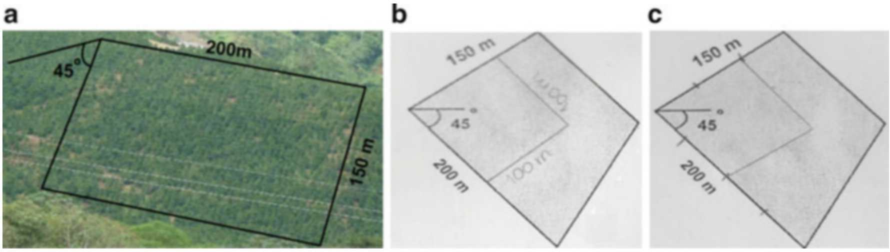


Fig. 19.1 Sloping field (**a**) and sketching process of Teams 1 (**b**) and 2 (**c**)

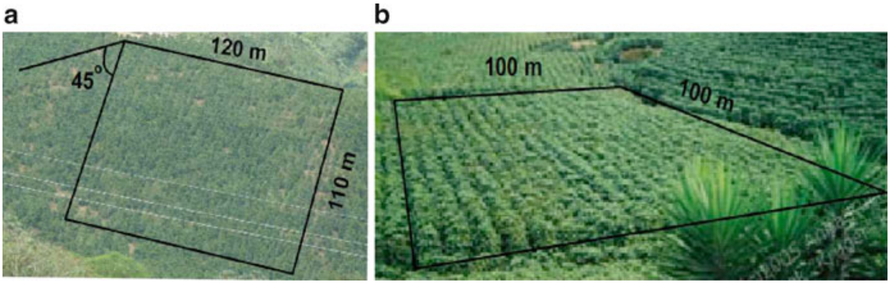


Fig. 19.2 Slope (**a**) and flat ground (**b**)

clear that the students noticed other variables that could be taken into account when making a decision regarding which land to buy in terms of advantages and disadvantages according to its topography, yet they only focused on the connection between the area and its relation to the amount of trees that could be planted.

According to students' answers, there is an apparent disconnection between mathematical considerations and cultural experiences. Accordingly, the teacher asked students to research how planting is done on both types of land. Students found in coffee farming books that the horizontal length from one tree to the other is the same in both plots of land (See Fig. 19.3a). The teacher suggested they make some drawings of the process. In one of the proposals, one of the teams had marked one metre lengths in both the hypotenuse and the side representing the flat land (see this mark in Fig. 19.3b). In this situation the teacher asked them to do it the way they would normally do it in their lands in their daily life. Immediately, students changed their layout and placed the trees according to tree projections (Fig. 19.3b). Later they stated:

Felipe: When the land is on a slope, we draw the squares with a one-metre distance, so as not to draw it one metre at ground level. [. . .] then we make a kind of reflection [projection]. One metre in the air, and then it is calculated one metre below, and it is repeated with the rest. When there is zero slope, it is marked every other metre. (Team 1)

Maria: The distance between each tree is measured; then, there is a metre and it is reflected [projected]. (Team 2)

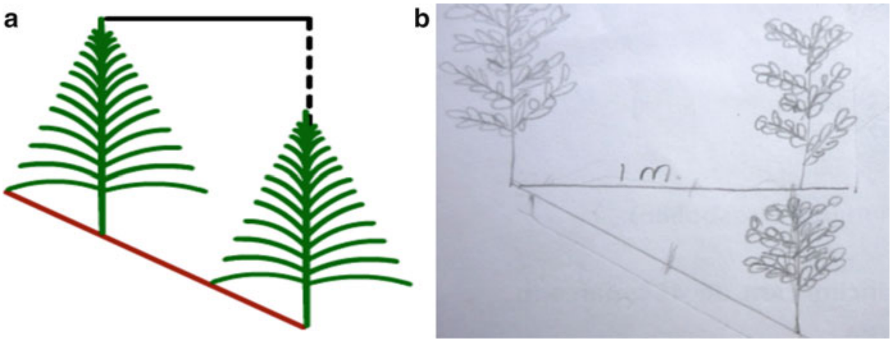


Fig. 19.3 Planting

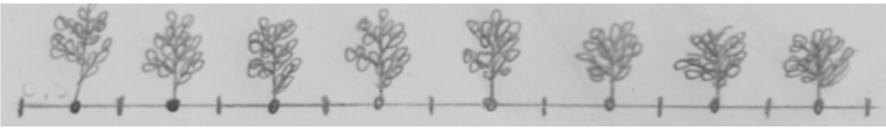
Next, the teacher suggested the students consider an 8-m distance to plant trees and to assume a diameter of one metre at the top of the tree. With this task, the two teams worked without any trouble and handed in a figure such as Fig. 19.4. Later, the teacher asked them to use the same length to plant trees on a sloping ground; in this same situation team 1 used a different argument, which is shown in Fig. 19.5.

The analysis of Team 1 students is not what the teacher asked for, but it allows us to notice that students also managed to differentiate the "Euclidean surface" from the area suitable for planting. In Fig. 19.5, it seems that the students used the two distances (flat and sloping) with the same size; thus, projecting the trees from the flat land to the slope. That is, students sketched the trees in the sloping land considering the same angles as in the flat land. So the teacher asked them; how many trees fit on the sloping land? Felipe replied:

> . . . well, there are two ways. If we plant considering the projection as we have always done, we plant 6 trees [taking into account the sloping land is 8 metres], because they are the projection, [. . .], it is almost possible to plant seven trees, but actually the seventh one is not possible as it is necessary [to have] an additional piece of land.

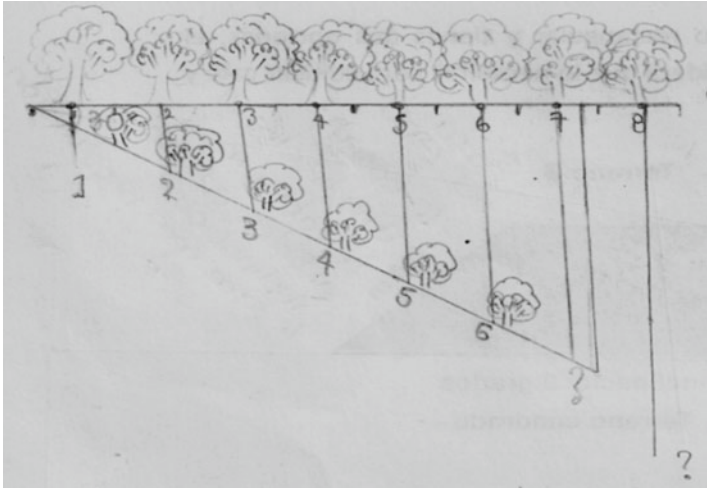
In his conclusion Felipe said: "if you want the same number of trees in a sloping plot of land as in a flat one it is necessary to have a longer area".

These findings allow us to see that students were able to overcome the idea they had about a "Euclidean surface" as an equivalent to "agricultural area" concluding that beyond physical distances, all surfaces having minimum horizontal projections are equivalent in agricultural terms (e.g., Fig. 19.6a, surfaces on natural distance, geometric distance and reduced distance have the same tree capacity despite having different physical length). When students were able to understand that a slope is a determining feature in the number of trees that would fit in a plot of land, they were able to produce some mathematical models for flat plots of land (Fig. 19.6b) and for plots of land having a sloping land projection (Fig. 19.6c).



Conclusion: 8 trees fit in 8 metres leaving a one-metre distance from tree to tree

Fig. 19.4 Horizontal planting (Team 2)



Conclusion: 5 trees can be planted in a plot if they are planted considering their reflections [projections].

Fig. 19.5 Planting on a sloping terrain (Team 1)

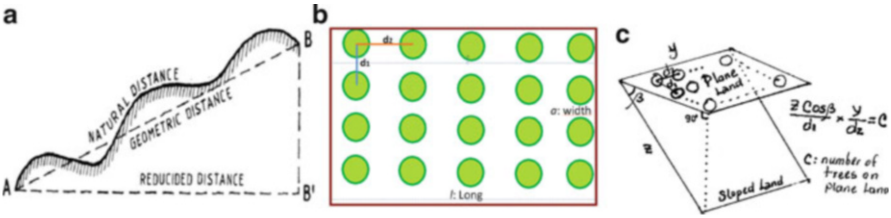


Fig. 19.6 (**a**) Three types of distances (Berrío 2012) (**b**) Model of number of trees $C = al/(d_1 d_2)$ (**c**) Model of number of trees on both lands

19.5 Discussion

International research has shown that (figurative) contexts can have very individual, unpredictable effects (Busse and Kaiser 2003) and that in an authentic context, students not only participated and were empowered with such aspects as gathering data, producing models and their meanings, but also, become more aware of phenomena related to the aforementioned context (Muñoz et al. 2014). In the student cases reported in this chapter, tasks arose from student-teacher discussions and agreements; in other words, tasks did not appear as a priori *situations* structured by the teacher.

The data presented above show that at first students believed that a “Euclidean surface” and “arable land” were equal. This fact may be interpreted as an apparent disconnection between the world students live in and contextual situations described in classroom tasks (Stillman 2000). To do an in-depth study to understand this apparent disconnection, students were asked to describe why they believed so. Students argued that “the larger the area [geographical or ‘Euclidean’] the greater its tree capacity” in this case, students did not take into account the slope in the plot of land. In fact, some students pointed out that coffee-growers believe that to have a sloping field is to have more [geographical] area; therefore, it can be sold for a higher price. According to students’ statements, beliefs such as the above-mentioned can be considered typical of the coffee growing culture; thereby, aspects such as explanation systems, philosophies, theories and daily actions and behaviour must identify with culture (D’Ambrosio 2005).

The findings reported in the previous section show that this notion of equivalence between surfaces did not remain stable during the modelling development. On the contrary, such an idea changed as other thoughts emerged. Thus, students’ reflective attitude became a revitalizing action without ignoring the complexity of the phenomenon studied; it allowed students to extend their ideas and thoughts on the phenomenon being studied.

A fact worth mentioning has to do with the situation shown in Fig. 19.4, where, in one of the groups, the students tried to use the same measure between the bases of the trees in both lands. This action seemed to be inconsistent with the daily procedures carried out in their culture, which measures the distance using only horizontal figures. This action was a consequence of the previously explored idea about the equivalences between the areas; in this situation, this idea seemed to overshadow one of the daily procedures on which coffee tree planting is based. Thus, it presented a certain “lack of meaning” of the mathematical activity at school. According to this, the teacher-researcher motivated the students to support their classroom actions and knowledge built from their everyday activities. This way, the task is reoriented, and students can recognise the meanings associated with context, in other words, it becomes a tool to understand the world.

Accepting the ideas students have on certain cultural aspects and not remaining neutral in a mathematical modelling process requires a recognition of the role of context as an establishing element of scientific knowledge at school (i.e., in mathematics, other disciplines and the context itself). This does not only have a utilitarian purpose, that is, so situations in which it is used as a context can promote mathematical content or motivate students to mathematics; nor, once knowledge is obtained, is context set apart solely for focusing on the mathematical knowledge that could have been derived from the situation. In other words, the main role of context in mathematical modelling must be to open the possibility of providing a cultural system of reference for mathematical activity, which gives value to pedagogical modelling which is not only limited to a strategy or a teaching method, or the view of mathematics *set* in a context (mathematics-in-context). However, it does not exclude these elements.

The previous ideas challenge researchers and teachers on the development of skills that allow them to focus not only on the aspects that lead to mathematical model production, but also to recognise other aspects, as shown in this chapter, arising from students' interests. Likewise, they demand a certain sensitivity to recognise the opportunities that a context offers both to include student development skills and to create knowledge based on context. To a certain extent, it is to develop an idea about reality itself (Villa-Ochoa and López 2011).

19.6 Conclusions

This chapter is based on a mathematical modelling thread in which students became involved, supported by a teacher-researcher, in the understanding of a typical situation of their own culture. Thus, the results of this study are consistent with those provided by Rosa et al. (2012) who stated that in mathematical modelling the knowledge was context based as it derived from experiences and it was strengthened by the cultural meanings in which the people were immersed. However, beyond the mathematical knowledge students made use of mathematical modelling (trigonometric models, rational functions, variations and measurements, etc.) in this study, this chapter shows that students' cultural or context knowledge does not remain static throughout a modelling activity. In particular, we show that there are situations in which mathematical modelling allows some considerations, ideas, beliefs, and explanations that are part of students' culture, and we want these situations to be reconsidered and rethought at least at an individual level.

According to D'Ambrosio (2009), over time we would expect individual knowledge to be discussed and analyzed from the perspective of its compatibility until achieving socially shared knowledge. In that sense, this research states that knowledge shared by the group is then socially organized, thus becoming a body of knowledge which is a response to its members' needs and will. In the situation mentioned in this chapter, it could be stated that mathematical modelling activated other dynamics of the individual knowledge of some members of a culture.

The research, from which this chapter derives, found evidence that converges with other studies that highlight the educational and social role in modelling (Barbosa 2006; Araújo 2009). In addition, this study highlights the intention not only to bring a context or situation from culture with motivational purposes, or to introduce or produce a concept or to produce utilitarian ideas for mathematics by showing it is everywhere, but also it has many applications. Hence, without it, scientific knowledge would not have reached its current level of development. This is not just about learning a specific content in context or developing skills to identify "*forms*" from a context comparable to mathematical "*forms* ". In contrast, it is to accept the role of non-mathematical knowledge that emerges in a modelling process. This research encourages researchers to produce other studies showing implications to promote situations in which mathematics and context are linked without being subordinated to each other.

References

Araújo, J. D. L. (2009). Uma abordagem Sócio-Crítica da modelagem matemática: A perspectiva da educação matemática crítica. *ALEXANDRIA Revista de Educação em Ciência e Tecnologia, 2*(2), 55–68.

Barbosa, J. C. (2006). Mathematical modelling in classroom: A socio-critical and discursive perspective. *ZDM, 38*(3), 293–301.

Berrío, M. (2012). *Elementos que intervienen en la construcción que hacen los estudiantes frente a los modelos matemáticos. El caso del cultivo de café*. Unpublished master's degree dissertation. Universidad Nacional de Colombia.

Blomhøj, M. (2004). Mathematical modelling - a theory for practice. In B. Clarke (Ed.), *International perspective on learning and teaching mathematics* (pp. 145–159). Göteborg: National Center for Mathematics Education.

Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45–58.

Borba, M., & Villarreal, M. (2005). *Humans-with-media and the reorganization of mathematical thinking*. New York: Springer.

Busse, A., & Kaiser, G. (2003). Context in application and modelling—an empirical approach. In Q.-X. Ye, W. Blum, S. K. Houston, & Q.-Y. Jiang (Eds.), *Mathematical modelling in education and culture: ICTMA 10* (pp. 3–15). Chichester: Horwood.

Christiansen, I. (1999). Reflexiones críticas sobre modelos matemáticos en la clase: Sueño o realidad? *Revista EMA, 5*(1), 29–50.

Creswell, J. W. (2008). *Educational research. Planning, conducting, and evaluating quantitative and qualitative research*. Upper Saddle River: Pearson, Prentice Hall.

D' Ambrosio, U. (2005). Sociedade, cultura, matemática e seu ensino. *Educação e Pesquisa, 31* (1), 99–120.

D' Ambrosio, U. (2009). Mathematical modeling: Cognitive, pedagogical, historical and political dimensions. *Journal of Mathematical Modelling and Application, 1*(1), 89–98.

Ministerio de Educación Nacional. (2006). *Estándares Básicos de Competencia*. Bogotá-Colombia: Cooperativa Editorial Magisterio.

Muñoz, L., Londoño, S., Jaramillo, C., & Villa-Ochoa, J. A. (2014). Contextos auténticos y la producción de modelos matemáticos escolares. *Revista Virtual Universidad Católica del Norte, 42*, 48–62.

Rosa, M., Reis, F., & Orey, D. (2012). A Modelagem Matemática Crítica nos Cursos de Formação de Professores de Matemática. *Acta Scientiae, 14*(2), 159–184.

Stake, R. E. (2007). *Investigación con estudio de casos*. Madrid: Morata.

Stillman, G. (2000). Impact of prior knowledge of task context on approaches to applications tasks. *The Journal of Mathematical Behavior, 19*(3), 333–361. doi:10.1016/S0732-3123(00)00049-3.

Villa-Ochoa, J. A., & López, C. M. J. (2011). Sense of reality through mathematical modelling. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 701–711). New York: Springer.