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# Teaching and Learning Mathematics in the Digital Era: Challenges and Perspectives

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Abstract: In 1985, the first ICMI Study was entitled « The Influence of Computers and Informatics on Mathematics and its Teaching ». Twenty years later, ICMI decided to launch a new Study on technology « Mathematics Education and Technology – Rethinking the Terrain » whose results were published in 2010 in a volume of the Springer NISS Series. In the first part of the text, relying on these two studies and my personal experience in the field, I briefly review the history of technology in mathematics education, pointing out that we have now entered a new era, what means new educational conditions, resources, affordances and responsibilities. I question then the potential of the knowledge accumulated in this area for taking up the challenges we currently face, trying to make clear that even if the world is changing, the knowledge we have gained through research and practice is certainly limited but still highly pertinent and useful.

## I. Introduction

This is not by chance that the first study launched by the ICMI (International Commission on Mathematical Instruction) in 1985 was devoted to « The Influence of Computers and Informatics on Mathematics and its Teaching » (Churchhouse 1986). Those in charge of the Commission were already convinced that the world was changing and that the emerging computer sciences would deeply impact as well mathematics as mathematics education. The dynamics anticipated were not exactly those expected, however progressively technological tools and values entered educational systems, influencing mathematics curricula, teaching and learning processes. Research and innovation developed, accumulating knowledge. Twenty years after this first study, the ICMI Executive Committee considered that the time had come to capitalize on this experience for thinking about the future. A second study was thus launched entitled « Mathematics Education and Technology – Rethinking the Terrain ». This study, whose conference took place at the end of 2006 in Vietnam (Hoyles, Lagrange, Le Hung Son and Sinclair 2006), and whose resulting volume was published in 2010 (Hoyles and Lagrange 2010), made clear the state of the art both in terms of research advances and practices at the end of the first decade of the twenty-first century. We are now just a few years later. However, new technologies still marginal at the time of this study such as smart phones are spreading at an incredible speed over the world, and today tablets and the Internet are becoming the way many children enter the digital world from their early ages. We today speak of "digital natives" for qualifying our pupils and students, who spent so much time playing electronic games, sending SMS and communicating through social networks. The French philosopher Michel Serres calls them "Petits poucets" and "Petites poucettes" due to their capacity to send SMS using their thumbs (Serres 2012). New educational practices have emerged such as the so-called MOOC (Massive Open Online Courses) and they quickly spread out in the university world. We can no longer deny that we live in a digital era, which deeply affects our personal and social life, our working and learning modes. We can no longer consider that teaching mathematics with technology is just an option. In this changing world, up to what point does the knowledge that has been accumulated through research and experience in mathematics education since the first ICMI Study allow us to take up the new challenges we meet? How can we best use it and what more is needed? These are the main questions addressed in this text. Relying on the two ICMI Studies and on my personal experience in the field, I first briefly review the history of technology in mathematics education, pointing out that entering a new era means new educational conditions, resources, affordances and responsibilities, thus new challenges to take up. I discuss then the potential of the knowledge accumulated in the area for taking up these challenges, defending my conviction that, even if the world is changing, the knowledge we have gained through research and practice in the three last decades is highly pertinent and useful. I will end by evoking some research and development projects I have been recently involved in that reinforce this personal conviction.

## II. Digital technologies and mathematics education: an historical vision

The history of the relationships between what we called today digital technologies and mathematics education is a long history that can be traced back to the emergence of computer sciences. Among the school disciplines, mathematics is indeed the one that has been influenced the first by this scientific and technological development. The role played by mathematicians in the emergence of computer sciences, and the resulting proximity between the two disciplines, certainly contributed to this situation. It also impacted the way digital technologies entered the world of mathematics education, for instance through the importance attached to the idea of algorithm, a fundamental idea in computer sciences, and to programming activities. The first ICMI Study mentioned above, and entitled "The Influence of Computers and Informatics on Mathematics and their Teaching", well reflects this close connection and influence. It was launched at a time when there was already a strong consensus for estimating that both the technological evolution and the computer sciences (informatics in the Study) were likely to deeply impact mathematics teaching and learning. It situated also at a time when some mathematicians still needed to be convinced that this evolution would substantially impact their practices and their discipline, as was expressed by Jean-Pierre Kahane who was the President of ICMI in 1985, in the interview realized for the ICMI Centennial in 2008[1].

The main technologies mentioned in this study were scientific calculators and mathematics computers software such as MACSYMA with symbolic capabilities; only a few contributions explored the potential of the new graphical affordances of computers. Already at that time experiments had been carried out with children and younger students, as proved by the work initiated by Papert with the Logo language and the Turtle artifact (Papert, 1980), but the Study only covered university and pre-university levels, and many contributions focused on the teaching of Calculus. In fact, several contributions were just prospective texts without experimental base, and the

[1] The video of this interview is accessible on the historical web site of ICMI http://www.icmihistory.unito.it

experiments reported were all small-scale experiments. Technological tools were expected to foster the introduction of experimental approaches in mathematics teaching and to allow students to focus more on conceptual issues, not just on the learning of techniques; programming activities, often made necessary by the limited affordances of the technology used, were expected to contribute to such a conceptual understanding. Curricular changes were asked for, especially less emphasis on the mastery of calculations than calculators and computers could easily take in charge, and also more importance attached to domains such as discrete mathematics. Another important point to mention is that this study made clear, through the contributions it gathered, that digital technologies were still at that time the privilege of developed countries.

UNESCO published an up-date version of the Study a few years later (Cornu and Ralston 1992). In the meantime, the number of experiments had substantially increased reinforcing the conviction that digital technologies had substantially to offer to mathematics education. However, in the overview introducing the volume preface Burkhardt and Fraser wrote:

> "Nearly all the chapters that follow make suggestions for new curriculum elements based on these new methods of doing mathematics; readers will find many of these arguments stimulating and even persuasive. Changes are surely needed and these suggestions seems better grounded than most.
> Nonetheless, it must be recognized that such suggestions are fundamentally speculative at the level of large-scale implementation – by which we meant that converting them into a well-developed and tested curriculum for the typical teacher and the typical student is still a major challenge." (Cornu and Ralston 1992 p.3)

When the second ICMI study was launched in the early 2000, the situation had substantially changed. As pointed out in the introduction of the associated volume:

> "Since the first ICMI Study in 1992, there have been major developments in digital technologies in terms of hardware: computers of all types, calculator and handheld technologies, digital technologies widely used in society at large such as mobile phones and digital cameras, and of course the massive influence of the World Wide Web. Aligned to these hardware changes, new software have been developed with potential impact on all phases of education, and on informal contexts of education. By the time of ICMI Study 17, digital technologies were becoming ever more ubiquitous and their influence touching most, if not all, education systems. In many countries, it is hard to conceive of a world without high-speed interactivity and connectivity." (Hoyles and Lagrange 2010, p. 2)

The situation had also changed because many mathematics curricula had promoted the use of digital technologies and even in a substantial number of countries made it compulsory (see for instance (Artigue, 2011) for an analysis of the French case). As a consequence, implementation had become more systematic and not just reserved to local experiments in ecologically protected environments; specific courses had been organized for teachers. It had also changed because researchers in mathematics education had strongly invested in the area, developing theoretical frameworks and constructs, accumulating experience and results regarding the design of digital artifacts, the precise affordances of different technologies and the ways these could be actualized in classrooms, learning processes in digital environments, teachers' practices, preparation and professional development, curricular change… They had been involved in the piloting of large-scale projects. Last, but not least, the situation had changed because, when the second ICMI Study was launched, digital technologies were no longer reserved to developed countries. They impacted societies and educational systems all over the world. In the meantime, moreover, research had become more sensitive to equity issues and to the danger that, instead of serving the cause of quality mathematics education for all, technology might contribute to new disparities in learning opportunities.

The organization of the Study itself reflected this evolution. Seven themes were set out in the associated Discussion Document: (1) Mathematics and mathematical practices, (2) Learning and assessing mathematics through digital technologies, (3) Teachers and teaching, (4) Design of learning environments and curricula, (5) Implementation of curricula and classroom practices, (7) Connectivity and virtual networks for teaching. It is interesting to notice that the first and the last one had few contributors. As pointed out in the introductory chapter (p. 4), the lack of contributions proposed on the theme of mathematics and mathematical practices made clear that, contrary to the first Study, this issue was no longer a matter of debate, while the lack of contribution on the last theme certainly reflected the limited research existing at that time on dimensions of the digital era that are now so pervasive. On all the other themes, the different chapters of the resulting volume show the diversity and richness of the contributions on which the Study was able to capitalize for building a coherent view of the state of the art and showing the advances achieved since the first Study. I will come back to these in the next section.

More especially, if we consider the themes of reflection proposed for this HTEM Conference, we observe that all of these are focus points in one chapter or another, at the exception of the integration of history of mathematics in the classrooms with the use of technology. In section 1 entitled *Design of Learning Environments and Curricula*, one chapter for instance analyzes and compares five projects developed in different countries for implementing digital technologies at a national scale: *Enciclomedia* in Mexico, M@t.label in Italy, *Sketchpad for Young Learners* in the US, *Mathematics 9 and 10 with the Geometers's Sketchpad* in Lithuania, and *E-content inititative* in Iran. In section 2 entitled *Learning and Assessing Mathematics with and through Digital Technologies*, Chapter 10 especially deals with assessment supported by digital technologies, and Chapter 11 with the collaborative and participative visions of teaching and learning that are promoted by digital technologies. Four chapters are devoted to teachers and technology addressing both teachers' practices and teacher education courses in mathematics and technology. Issues of access and equity are especially addressed in section 4 entitled *Implementation of Curricula: Issues of Access and Equity*. Even the theme of connectivity and virtual networks for learning was eventually adressed through the organization of a specific pannel at the Study Conference, the contributions to this pannel being collected in Chapter 22 of the section 5 entitled *Future Directions*. Last but not least, issues regarding the design and management of tasks involving digital technologies are dealt with thorough the whole book. However, despite the evident progression of knowledge and use of digital technologies, the reading of the ICMI Study volume also makes clear the initial ambition of improving mathematics teaching and learning through the large-scale implementation of digital technologies is still to be achieved, and that making a productive use of technological affordances both for teaching students and for teacher education remains a problematic issue.

This being said, as pointed out in the introduction, there is no doubt that since the second ICMI Study was carried out, the world has going on changing. Digital technologies impact more and more in a dramatic way all aspects of the personal, social and professional lifes of people, those of students as those of teachers. New technologies are spreading at an incredible speed. More tablets than computers are sold today and they offer an increasing number of mathematics applications. Not to mention the huge number of applets and video clips produced as educational resources and easily accessible, even now on cell phones. Tactile screens are becoming the standard format of graphical interfaces, creating news ways for acting on representations of mathematical objects and processes. Social networks and collectives exponentially increase, becoming more and more influential; they deeply impact our communication modes, and even our relationships with authorities and institutions. Programs of education at the distance multiply and take massive forms as is the case with MOOC. There is an evident acceleration of the digital era, taking a diversity of forms.

All these changes impact directly or indirectly learning processes and the teacher work in the classroom and outside the classroom, the relationships between teachers and students, between the schools, their environment and the society at large. They create new affordances, but also new challenges and new responsibilities. How to avoid an increasing divorce between social practices and school practices, with the resulting negative effects on the image of the discipline and the students' motivation for engaging in its study? How to benefit in mathematics education from the incredible amount of accessible information and resources? How to benefit from the changing modes of social communication and Internet facilities for creating and supporting communities of mathematics teachers, and definitively break with the vision of the teacher as a solitary worker? How to use the new affordances for giving students more autonomy, for supporting inquiry-based practices, for developing a mathematics education more open on the outside world and its challenges while being faithful to the epistemology of the discipline? How to use these new affordances for addressing better the specific needs that our students may have? How to make that these new affordances benefit to all and do not exharcerbate inequalities and discriminations? I will stop there without pretending to have covered in any way the different challenges that we currently face. They are many.

We have also new responsibilities. More than ever, school is not the only source of information for our students. When a question is raised, a problem set up, relevant information or even answers already elaborated are more and more immediately accessible on the Internet, and human beings have developed a new reflex often called "the Google reflex". However learning mathematics is not achieved just by accessing mathematical information or even direct answers on the Internet. Accepting to live in the digital era is accepting to have this flux of information entering the school world but also to instaure what Chevallard's calls appropriate *dialectics between media and milieux* (Chevallard, 2007). What is got through the Internet are just media. This is our teacher's responsibility to create milieux from these media, with the meaning attached to this term in the theory of didactical situations (Brousseau, 1997), allowing the students to build mathematical knowledge from these media. It is our responsibility to help them cultivate the critical stance necessary for using these media in an appropriate way, in and outside the school world. Mathematics education has also a social responsibility in making visible some hidden parts of the technologies upon which we are becoming so dependent in this digital era. For instance, the last reform of the French high school curriculum has introduced in the mathematics speciality in grade 12 themes of work around matrices. An exemple of application presented in the accompanying documents and used in all textbooks is "How Google works". This is indeed a very interesting example showing the power of mathematical abstractions such as Markov chains[2]. But we should not consider that this topic is only a good opportunity for introducing Markov chains and having students calculate (with calculators or computers) power of matrices, experience the stabilization phenomenon and understand how this can be used for ordering web pages. We should also discuss the consequences of such a functioning and how it can be and is currently used and perverted. I will stop there, the purpose of this paragraph being just to stress that new affordances, new conditions always go along with new responsibilities, and hoping that the HTEM Conference will be an opportunity for discussing this issue and the specific forms it may take in the region.

## III. The potential of accumulated knowledge for taking up the current challenges

As claimed in the introduction, even if we face new challenges, we do not start from scratch. The rapid evolution of digital technologies, the new affordances offered, makes it tempting to consider that this new digital era needs fresh minds and ideas, and that the experience gained with the "old" digital technologies may be not so much helpful. This is not my position. We have worked for decades for making technological advances serve the cause of mathematics education; we have experienced the difficulties of the enterprise and we have learnt from these difficulties. The new technological affordances do not make obsolete the technological artifacts that have been patiently elaborated such as for instance the many dynamic geometrical systems (DGS in the following), and the knowledge we have accumulated through their use in very different contexts. The new affordances will not miraculously remove the difficulties that educational systems have at benefitting from technology, and will not miraculously solve the problems met by teacher education and professional development. Thus, for instance, researchers having worked for a long time in the field do not necessary align with the discourse developed around MOOC in some quarters, even if they find it interesting to explore the potential that these new technologies offer. There is more continuity than discontinuity in the trajectories of educational systems whose dynamics obey complex processes influenced by many factors and by their history. Moreover, as evidenced by the ICMI Study 17, research in mathematics education is not disconnected from these evolutions (see for instance the different projects evoked in chapters 11 and 22, and dealing with collaborative and participative learning through the establishment of communities of practice and inquiry using web facilities and connectivity).

[2] See for instance the vignette written by Christiane Rousseau on this theme on the blog of the Klein project of ICMI : blog.kleinproject.org

Evaluating the potential and limitation of accumulated knowledge for taking up these challenges is thus especially important. While trying to contribute to this evaluation, I do not pretend to be either exhaustive or objective. My discourse is obiously shaped by my personal experience, necessarily limited (for instance I was never involved until now in distance learning projects) and by the different institutions through which I have met technological issues. Beyond these limitations, the space open to a lecture obliges to make a drastic selection. After reflection I decided, for this opening lecure, to develop a rather general view, emphasizing some constructions and research orientations that I feel especially promising. I have no doubt that the different contributions at the HTEM Conference will offer complementary perspectives and a lot of insightful examples.

In the ICMI Study 17, a specific chapter (chapter 7), co-authored by Drijvers, Kieran and Mariotti with the support of eight contributors to the ICMI Study Conference, is devoted to the theories and constructs that have been used, adapted for, or especially developed for the study of learning and teaching processes in mathematics in digital environment as well as for supporting the design of digital artefacts. The authors review the development of these constructions from the time of the first ICMI Study, pointing out that they have been influenced both by the technological evolution and by the global evolution of the field of mathematics education and, especially, the move from constructivist to socio-cultural theories with the consecutive increasing attention paid to the semiotic and communicative dimensions of learning processes.

### III.1. First theoretical constructs and research orientations

Evoking the early times of research in the area of digital teachnology, the authors evoke successively:

- the distinction made by Taylor (1980) between three different roles given to the computer: *tutor*, *tool* and *tutee*[3];
- the distinction between considering technological tools as *white boxes* or *black boxes*[4] introduced by Buchberger in the context of CAS use (1990), a dichotomy further refined for instance by Cedillo and Kieran (2003) in their development of *grey-box* teaching approaches;
- the well-known elaboration by Pea (1987) of digital tools as *amplifiers* and *reorganizers* of mathematical thinking
- and of course the *constructionist* approach of "learning by making" in carefully designed digital environments, initiated by Papert in the Logo turtle environment since the seventies (Papert, 1980).

[3] To use the computer as a tutee is to tutor the computer. This is the case when the student programs the computer for making it perform some task for her.

[4] Students use technology as a white box when they know the mathematical techniques that they ask the technologies to carry out for them. If they do not know how the results are produced, they use it as a black box. Buchberger thought that the use of CAS as a black box might be detrimental for learning processes.

Following the historical development of research in this area, several theoretical developments are then presented with more details. The approach developed by Noss and Hoyles in terms of *webbing* and *situated abstraction* is the first one:

> "The idea of webbing is meant to convey the presence (in the digital setting) of a structure that learners can draw upon and reconstruct for supporting – in ways that they choose as appropriate for their struggle to construct meaning for some mathematics." (Noss and Hoyles, 1996, p.107).
> Situated abstraction "describes how learners construct mathematical ideas by drawing on the webbing of a particular setting which, in turn, shapes the way the ideas are expressed (ibidem, p. 122).

As pointed out by the authors of the chapter:

> "What is interesting, and extremely relevant, about the approach taken by Noss and Hoyles is the way in which they took two theoretical concepts from the general mathematics education literature, that is, *scaffolding* and *abstraction*, and reworked them so as to better capture some of those aspects that are special about learning in technological environments, as well as how this learning can spark connections with other mathematical settings." (Drijvers, Kieran and Mariotti 2010, p. 100)

What is also interesting is the way this theoretical construction, which fully acknowledges the situated and social character of learning, has inspired design activities such as developed within the *Playground* and *Weblabs* projects, also mentioned as pionnering projects in terms of use of connectivity affordances in chapter 22.

The way the concept of milieu, with the specific meaning it has in the theory of didactical situation as a system antagonist to the learner (Brousseau, 1997), has been fruitfully used by different researchers for designing tasks in digital environements is then analyzed, before coming to approaches focusing on the way digital technologies can support the perceptuo-motor dimension of mathematical learning with reference to well known research in that area, such as that developed by Nemirovski and Rasmussen around the Water Wheel artifact. Reference is also made to the metaphor of Humans-with-Media introduced in (Borba and Villarreal 2004) to challenge the usual separation made between humans and the technologies they use, and stress that:

> "humans are constituted by technologies that transform and modify their reasoning, and, at the same time, these humans are constantly transforming these technologies". (Borba and Villarreal 2004, p. 22)

The interest of such a vision in the current context cannot be denied.

The theoretical landscape presented in the first sections of this chapter is thus already quite rich in approaches and constructs. However, this is not the end of the story as the authors come then to two different theoretical perspectives that were taking increasing importance in research in the area at the time the Study was carried out, as reflected in the number of contributions referring to these perspectives at the Study Conference: *instrumental and semiotic perspectives*.

### III.2. Instrumental perspectives[5]

At the previous HTEM Conference in 2008, I already evoked the instrumental approach that emerged in France in the mid-nineties and to which I contributed (see for instance (Artigue, 2002, 2011)). I will just summarize its main characteristics below before envisaging some recent developments that I find especially promising. This approach combines the affordances of Rabardel's vision of instrumentation in ergonomy and those of the Anthropological Theory of Didactics (ATD in the following). It emerged in the context of research about the integration of CAS technology, but its interest was then evidenced for dealing with different types of digital technologies from Spreadsheets and DGS to tutorial software. It shares with the other instrumental perspectives the essential distinction made between *artifacts* and *instruments* (Rabardel, 1995, 2002). An artifact denotes here the digital technology or one part of it that is used as a tool (a CAS calculator or its symbolic application, an Internet resource, etc.); an instrument is what this artifact becomes for a user who purposely uses it to carry out some type of task. An instrument thus is made both of an artifact and of the techniques and/or schemes that the user has developed for carrying out tasks with it. A fundamental idea in Rabardel's construction is that of *instrumental genesis*. Instrumental genesis denotes the process through which an artifact becomes an instrument for a given user which may be in our case a student, a teacher, but also a collective (for instance the process through which a spreadsheet software becomes an instrument for a student for studying sampling fluctuation in statistics, or the process through which the TI-Navigator system or other similar system become a collective instrument for the teacher and students of a particular classroom, for connecting different representations of functions). This process is bidirectional[6] what is expressed by the distinction made between *instrumentalization* (denoting the way the user shapes the tool for its own use) and *instrumentation* (denoting the way the user's strategies and knowledge are shaped by the tool). For a long time, research regarding digital technology has under-estimated the complexity of instrumental geneses and up to what point these geneses shape, not only how we learn mathematics in digital environments but also the mathematics we learn. Instrumental approaches have made this complexity visible. In the instrumental approach I contributed to, Rabardel's construction is combined with ATD, an anthropological approach, making clear that knowledge emerge from practices that are institutionally situated (Chevallard, 1992, 1999). Among many other constructs, ATD offers a model of human practices in terms of *praxeologies* that we found very promising for research in the area. Praxeologies indeed have both a *practical block* made of a *type of task* and a *technique* for carrying out the task (for the most elementary praxeologies), and a *theoretical discursive block* whose function is to describing, explaining and justifying the technique, what Chevallard calls the *technological discourse*, backed up itself by some *theoretical discourse* which may be more or less developed and explicit. This model resulted especially useful for overcoming the fallacious opposition made between conceptual and technical activity in many texts

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[5] As made clear in the Study chapter, Rabardel's ideas about instrumentation have been used by researchers in mathematics education having a diversity of educational cultures and background theories. This has led to a diversity of instrumental perspectives sharing some essential elements but differing according to the way Rabardel's ideas are combined with the backgroung theories they rely on.

[6] Note that this dialectic vision is in line with that mentioned above underlying the metaphor of humans-with-media.

regarding the affordances of digital technologies for mathematics teaching and learning. It helped us make clear and address the double function of instrumented techniques: a *pragmatic function* through the way they contribute to the production of answers, and an *epistemic function* through the way they contribute to the understanding of the (mathematical) objects they involve.

Through its different chapters, the second ICMI Study provides many examples of productive use of instrumental perspectives, this one or variants of it, for understanding learning processes in digital environments, for designing digital artifacts and for didactical design. It also shows different visions of the balance between instrumentalization and instrumentation processes. For instance, in chapter 13 devoted to collaboration between teachers and researchers, one of the case studies presents the collaborative work of Greek teachers and researchers in a context where Rabardel's ideas are combined with a constructionist approach. The case study shows how the use of *half-backed microworlds*, in line with constructionist views, leads researchers to give a major role to instrumentalization processes. In this project, half-baked microworlds, in fact, act as catalysts for instrumentalization, and this is through the instrumentalization that teachers engage into "genuine mathematical discussion and activity into the context of professional practice" and professionally develop (p. 303). This strategy constrats with most uses of instrumental perspectives that, on the contrary, focus on instrumentation processes.

What the ICMI Study volume also shows is an evolution of research focus within these perspectives from issues regarding students' learning to issues regarding teacher practices, preparation and professional development, in line with the global evolution of the field. This move started with the incorporation of the idea of *orchestration* to the instrumental approach by Trouche (2005), this idea being further refined in collaboration with Drijvers who established a typology of instrumental orchestrations (Drijvers, 2011). As pointed out above, instrumental geneses, even when they are considered at the level of an individual student, have a social and collective dimension. Students develop these "within the context of the classroom community, and a process of collective instrumental genesis is taking place in parallel with the individual geneses" (Drijvers, Kieran and Mariotti 2010, p. 112) and, I would add, intertwined with these. This is what the idea of instrumental orchestration aims at capturing, an instrumental orchestration being defined by *didactic configurations* i.e. arrangement of the artifactual environment, and *exploitation modes of these configurations*. In the last decade, however, the extension of the instrumental approach to the teacher has been dealt with in a diversity of ways, according to the theoretical background of the researchers. The chapters of the section 3 of the ICMI Study volume devoted to the teacher give several interesting examples of combination of an instrumental perspective with theoretical approaches in terms of distributed cognition or communities of practices, for organizing the professional development of teachers around the communal design of digital artifacts or of tasks for digital environments.

In the recent years, in my research team, the LDAR, the potential of the extension of the instrumental approach for understanding teachers' practices in digital environments has been especially studied. This work has been mainly carried out by combining the affordances of the instrumental approach with the double approach to teachers' practices (DA in the following). DA, which combines didactical and ergonomic affordances, was initiated by Robert and Rogalski (2002). It relies on Activity Theory and considers teaching as a profession especially demanding because the teacher works in an open and complex system. Teachers' practices are regarded as a coherent, complex and stable system. These practices, defined as anything that the teacher does as a professional inside and outside the classroom, are approached through five different but not independent dimensions: *cognitive, mediative, institutional, social and personal*[7] (Vandrebouck, 2013). For addressing teachers' practices with digital technologies, three different settings have been distinguished in terms of use (Abboud-Blanchard and Vandebrouck 2012). The first setting, the *private setting* regards the technological activities developed by teachers, individually or collectively, in their private sphere, not directly linked to their classroom activities; the second setting called *private professional setting* regards activities for the classroom but performed outside the classroom such as the design of classroom tasks involving such or such digital artifact: the third setting, the *public professional setting* regards the teachers' digital activities taking place in the classroom. In these three settings, digital technologies instrument the teacher's activity, and in the third one, this instrumented activity intertwines with the student's instrumented activity. The hypothesis is made that an effective professional use of technology by teachers supposes a synergy among these three settings. Moreover, inspired by recent development in cognitive ergonomy and professional didactics (Rabardel and Pastré 2005) and considering that teachers work in fact with systems of artifacts, my colleagues have extended the idea of instrumental genesis to that of *genesis of use* whose dynamics combine personal and professional instrumental geneses in the three settings mentioned above intertwined in complex ways. As explained in (Lagrange 2013), the idea of genesis of use transcend artifacts and their diversity, and it takes into account the coherence of practices proper to a particular teacher.

For analyzing the evolution of teachers in their actual use of digital technologies, the DA takes into account both the five dimensions of teachers' practices mentioned above and three levels for their organization; the *micro level*, which is that of automatisms and routines; the *local level*, which is that of the daily life of the classroom, where the teacher's plans face contingency obliging her to improvise and adapt her plans; the *global level*, which is the level of projects and scenarios, situated in a long temporality. This organization is submitted to institutional and social constraints as well as personal characteristics. The introduction of technology induces intertwined evolutions in these three levels and in their relationships. For instance, the analyses carried out with teachers beginning to use digital technologies have shown that the lack of specific routines and also of global vision integrating the use of technology, induces an over-load at the local level. These analyses have also allowed to identify some regularities in the reactions to such an over-load and evolutions which can be expressed in terms of dynamic moves between these three levels (Abboud-Blanchard and Vandebrouck 2012), (Abboud-Blanchard 2013). I will not enter into more details but there is no doubt that

[7] Summarizing, the cognitive dimension regards the teacher's choices related to the mathematical content, the tasks proposed to students and their organization ; the mediative dimension, the mediations and transformations she operates when implementing these choices in the classroom ; the three other dimensions regard determinants of teachers' practices situated at different levels : social, institutional and personal.

such constructions help us understand the complexity of technological integration, and also the limitations of teacher education programmes which highly under-estimate it.

The last enrichment of the instrumental approach that I would like to briefly evoke is not reflected in the ICMI Study volume. This is the extension of this approach to the documentary work of teachers initiated by Gueudet and Trouche (Gueudet and Trouche, 2009), (Gueudet, Pépin and Trouche, 2012). The instrumental approach leads them to distinguish between documents and resources, the documents being the analogous of the artifact, and the resources developed by the teacher from these the analogous of the instrument. Research about the documentary geneses of teachers emerged very recently but it questions the usual vision of "educational material and resource", and the dichotomy author/user, through making visible and object of study the transformative work operated by the teacher, and thus her contribution to the conception process. Such a vision is certainly promising, considering the current technological evolution and its impact on our documentary practices.

### III.3. Semiotic approaches

As announced above, the chapter 7 of the ICMI Study volume also pays specific attention to semiotic approaches, whose influence in the field of mathematics education at large has been increasing in the last decade. One acknowledged affordance of digital technologies is the new forms of representations of mathematical objects that they give access to, together with new means for acting on these representations and for connecting them in a dynamic way. For that reason, research in the area of digital technologies has played a substantial role in the reinforcement of semiotic approaches, and this specific attention could be anticipated. However, even more than is the case for instrumental perspectives, semiotic issues are not approached in a unified way. In the volume, specific attention is paid to the idea of *semiotic mediation* introduced by Vygotsky (1978) and its use for understanding how the use of a digital artifact to carry out a task may lead to the learning of a particular mathematical content, without under-estimating the crucial role of *cultural mediator* played by the teacher in such a process. As explained (p. 116):

> "the mediating potential of any artifact resides in the double semiotic link that such an artifact has with both the meanings emerging from its use for accomplishing a task, and the mathematical meanings evoked by that use, as recognized by an expert in mathematics".

Within this perspective, from the activity with the artifact, signs emerge with personal meanings rooted in the context of the artifact, but their evolution towards mathematical signs needs the intentional and organized activity of the teacher. This semiotic approach has been fruitfully used for understanding and exploiting the mediating potential of a diversity of artifacts, both digital and not digital (see for instance (Bartolini Bussi and Mariotti 2002), (Falcade et al. 2007)). Arzarello contributed to it with the idea of *semiotic bundle* (Arzarello 2006) that is certainly of interest considering the current complex intertwining of semiotic tools at stake in cognitive activities. There is no doubt that the current spreading of touch interfaces is also a challenging context that these semiotic approaches can help the community of mathematics education address, in connection with the perceptuo-motor perspectives mentioned above.

### III.4 Social learning theories

Another promising orientation of research considering current technological advances is that provided by social learning theories, and especially those viewing learning as a process of increased participation in a community of practice (Lave and Wenger, 1991). As pointed out by Beatty and Geiger, the co-authors of chapter 11 entitled *Technology, Communication and Collaboration: Re-thinking Communities of Inquiry, Learning and Practice*, social approaches entered the area of mathematics education research on digital technologies rather late, and for instance at ICME-9 in 2000, contributions referring to such theoretical approaches remained very marginal. There was no longer the case at the ICMI Study Conference in 2006, and since that time the digital evolution and that of social practices has reinforced the influence of these perspectives. In chapter 11, the two authors structure the presentation of research in that area according to the characteristics of the technology used:

- technology designed for both mathematics and collaboration, that is to say technologies such as those used in the WebLabs and Playground projects already mentioned; these offer facility for learners to work with mathematical concepts in a virtual environment that specifically includes a component designed for communication;
- technologies designed for mathematics but not specifically for collaboration; in that case, collaborative practices result from choices made both in the design and management of the tasks proposed to students or teachers;
- technologies designed for collaboration but not necessarily for mathematics such as the ICT tools used in distance learning;
- and finally technologies designed for neither mathematics nor collaboration.

The examples provided for these different categories show up to what point the forms of communities of practice and the forms that learning may take in these are dependent on the type of technology used, even if the authors, referring to (Healy, Pozzi & Hoyles. 1995) point out (p. 278) that, in all these studies, technology is conceived as a means of mediating social interactions “not by constraining action, but by providing a medium through which shared mathematical expression can be constructed”.

### III.5. Concluding comments

There is no doubt for me that these different approaches and the most recent developments not reported here, offer substantial theoretical background for copying with the new challenges we have to take up. Better connection between them is however needed for overcoming the impression of fragmentation that such a description may give and make it possible to link the research results obtained through these different approaches. This cannot be the task of individual researchers, but is more a collective responsibility to be taken in charge by the community of mathematics education. As shown for instance by the activities carried out at the CERME Conferences of the European Society for Research in Mathematics Education since 2005, this is more and more the case, and results begin to emerge which go beyond the sole area of research on digital technologies (see for instance Part XVI entitled “Issues and Practices in Networking Theories in (Sriraman and English 2010)).

## IV Reflecting on my specific journey with digital technologies

As announced in the introduction, in this last section, I will adopt a more personal stance, looking back at the main research and development projects I have been involved in during the last decade, and considering their potential connection with the challenges evoked so far.

### IV.1 Entering the world of tutorial resources through a regional project

When the second ICMI Study was launched, I was in charge of piloting a project with the Ile-de-France region in France, involving more than 5.000 students and 100 teachers. This region, the biggest in France, had decided to finance the access to online mathematical resources to grade 10 students living in poor social areas in order to compensate the limited support that these students received outside the school itself. The region had also decided that a university team would evaluate the project and the IREM[8] of the University Paris 7, which I directed at that time, was contacted. As I explained in (Artigue 2010), this project was challenging for us, due to its size and to the *tutorial technology* used, so different from the digital technologies we were used to work with, CAS, DGS or Spreadsheets. Despite the limited quality of these technologies both in terms of the tasks proposed and the forms of didactic interaction implemented, we accepted the challenge, considering their increasing use in schools and at the periphery of the educational system. These technologies were advertised as friendly digital tools allowing students to learn mathematics in an autonomous way and at their own pace at home; allowing teachers who wanted to support the personal work of their students to differentiate the tasks proposed to them according to their particular needs, and also to get easy feed back about their work, successes and failures for supporting them in an effective way. Some tools also included communication facilities at the distance.

We accompanied teachers and students during three years and we observed a quite different reality. For instance, the poor quality of the didactic interaction implemented made that autonomous work was possible only for a minority of students; the differentiation and monitoring of students' activity resulted especially costly for teachers. We developed specific tools for analysing such tutorial resources and their learning potential, combining didactic and ergonomic approaches and relying on the notions of *utility*, *usability* and *acceptability* used in AIED[9] research. Moreover, the regularities observed in the ways students and teachers instrumented these technologies were the starting point for the extension of the instrumental approach to teachers' practices mentioned in the section III, further developed in the GUPTEN project, a national project focusing on the geneses of teachers' professional use of digital artifacts (Lagrange, 2013). Looking retrospectively, my feeling is that the knowledge gained through this project carried out with ordinary students and teachers, and addressing this specific type of digital technology, not so much considered by research in mathematics education but socially used, is not deprived of interest for addressing the new challenges

[8] IREM : Institute of Research on Mathematics Teaching. Information available on the portal of the IREM network : www.univ-irem.fr

[9] AIED : Artificial Intelligence and Education

that we face. This is certainly the case in my country at a time the Ministry of Education in France makes the entrance of schools into the digital culture a main educational goal and, among other projects, envisages the use of digital tools for the interactive personal guidance of students living in disadvantaged social areas.

There is no doubt that, in the last decade, the quality of this type of technology has improved. However, considering the work that my team has been developing since more than one decade for developing computerized diagnostic tools in elementary algebra relying on the advances of didactical research in that area, and more recently for designing differentiated learning trajectories based on this diagnostic (Grugeon-Allys, Pilet, Chenevotot-Quentin and Delozanne 2012), and for making the whole available on line, thanks to the collaboration with the very dynamic association of teachers Sesamath[10], I am perfectly aware of the difficulty of the enterprise and thus of the limitation of the didactic interaction proposed in most of these tutorials.

### IV.2 Capitalizing and networking: the TELMA and ReMath projects

From 2003 to 2009, I was involved successively in two European projects which emerged from the Kaleidoscope network of excellence: TELMA and ReMath. These projects aimed at a better capitalization of the outcomes of European research regarding the potential and use of digital technologies for the learning of mathematics, ReMath focusing more especially on the semiotic potential of technology. The same six teams from four European countries were involved in these two projects. We soon discovered that the fragmentation of theoretical approaches was an obstacle to the capitalization aimed at, and we tried to find ways for limiting this fragmentation. Within the European research team TELMA, we began to develop both a specific metalanguage, the metalanguage of *didactical functionalities of a digital artifact* and of *key concerns*, and a specific methodology, that of *crossed experimentations*, for that purpose (Artigue 2009a). These conceptual and methodological tools were then refined and amplified in ReMath[11] for dealing both with the design of digital artifacts and their educational use. Thanks to these theoretical and methodological constructions, and relying on the important work of design and of cross-experiments carried out (six different digital artifacts were designed or enriched offering a diversity of semiotic innovative affordances and covering a diversity of mathematical domains; each team experimented two different artifacts, a familiar and an alien, in realistic contexts over a substantial time; all the processes were precisely monitored in order to make the design choices explicit and comparable across the contexts and didactic cultures…), we were able to develop a *shared theoretical framework* for addressing representation issues in research regarding digital technologies in mathematics education, to build a *networked theoretical landscape* making explicit relationships between the main theoretical approaches used by the six teams involved in the project, and to express these results in an accessible discourse (Artigue, 2009b). Once again, my feeling is that such a research is helpful for addressing the challenges we face, through its practical results but also through the theoretical and methodological constructs it has created for addressing the fragmentation of the field.

[10] Information accessible at www.sesamath.net

[11] Information accessible at remath.cti.gr

### IV.3 Working collaboratively for developing educational resources

In the recent years, I have also been involved in the collaborative development of resources for technological integration, in mixed teams of teachers and researchers. The idea of community of practice is of course at the core of these collaborative processes. The work began in 2006 with a national project around the new calculator TI-nSpire, the project e-colab[12]. Three different IREMs were involved, those of Lyon, Paris and Montpellier. The development of resources obeyed a cyclic process, the resources produced and experimented in one IREM being then possibly adapted and tested by another IREM, and the adaptations and results being systematically compared and discussed, in face to face meetings but also at the distance through the use of a platform especially designed for the project. The whole process resulted in three successive books for grade 10, 11 and 12 (Aldon, 2009, 2010, 2011). The collective design was informed by the instrumental approach described in section III. This is visible for instance in the attention paid to the interaction between the development of instrumental and mathematical knowledge. It was also informed by the theory of didactical situations and especially the attention paid to the design of situations having a substantial a-didactical potential, and to devolution and institutionalization processes. Finally, it was informed by a flexible vision of educational resources as objects whose design continues in use, in coherence with the extension of the instrumental approach to the documentary work of the teacher mentioned in section III.

This collaborative work extended then with the same IREM partners in a Comenius European project, involving 20 institutions, 10 university and 10 associated high schools in seven different countries, the EdUmatics project[13], from 2010 to 2012. The aim of the project was the development of modules for a European development course for teachers focusing on the integration of digital technologies in mathematics teaching, possibly implemented at the distance. Once again the design was collaborative and cyclic. Each module was taken in charge by two teams from two different countries (two universities and the associated high schools). A global structure for the different modules and scenarios was elaborated and discussed for each module at a first collective meeting. Then each couple of teams worked on the design of the module it had in charge, the classroom situations used in it being experimented in the two associated highschools. The modules were then made accessible on the project platform, and an external evaluation of the modules was organized involving two new teams for each module, according to a process collectively agreed. On the basis of this evaluation, the modules were revised and, when in their final form, translated in six different languages and made publicly accessible. Collaboration among the teams once again combined regular meetings and communication at the distance through the mediation of technology. The diversity of the educational contexts and mathematics curricula involved made the project especially challenging. For copying with this diversity, in the module our IREM had in charge on functions, we relied on the idea of family of tasks associated with an explicit set of didactical variables, whose possible values were expected to allow the teachers or teacher educators using the module to adapt it to their

[12] Information accessible at http://educmath.ens-lyon.fr/Educmath/recherche/archives/partenariat-inrp-08-09/e-colab

[13] Information accessible at www.edumatics.eu

particular context without loosing the essence its didactical potential. I will not enter into more details regarding these two projects, but once again the collaborative and cyclic way in which the design was approached, the conception of resources as flexible objects whose necessary adaptation to the specific context of the user must be anticipated and facilitated from the design phase, is not without interest for facing the challenge we still face of large-scale effective use of digital technologies.

However, what the discussions around this project made also clear for me is the absence of a clear corpus of knowledge that might constitute a shared reference for organizing teacher education and professional development in this area.

## V. In conclusion

In this text, I have briefly reviewed the history of mathematics education and digital technology from the first ICMI Study devoted to technological issues carried out in the mid eighties to the ICMI Study 17, which revisited this terrain twenty years later. This history makes clear that we are entering a new digital era, creating new conditions and affordances for mathematics education, and resulting in new challenges. It also makes clear that in the last two decades we have accumulated a huge amount of knowledge through research and innovation in that area, that we are substantially equipped, conceptually and practically, for taking up these new challenges and, even more, that we already began to do so. This being said, we cannot deny that we are still struggling for making digital technologies efficiently serve the cause of quality mathematics education for all. This is indeed an old challenge but, in what concerns it, we have progressed much less than was anticipated when the first ICMI Study was launched in 1985. The forms this challenge takes are moving and, today even more than ever, for taking it up we need to be imaginative, while keeping in mind the lessons of the past. We will not succeed without relying on the solidarities and collectives that we have progressively developed and without using the communicative affordances of digital technologies. I will end this text, as I did at the closing lecture at the ICMI Study 17 Conference in Hanoi, by expressing my conviction that the contribution of emerging areas of the world is today essential and will become even more in the future. Here, in Brazil and Latin-America, we are in such an area.

## References

Abboud-Blanchard, M., & Vandebrouck, F. (2012). Analysing teachers' practices in technology environments from an Activity Teoretical approach. *The International Journal for Technology in Mathematics Education*, 19(4), 159-164.

Aldon, G. (2009). *Mathématiques dynamiques. Activités avec la TI-n**spire** pour la classe de Seconde*. Paris: Hachette Education.

Aldon, G. (2010). *Mathématiques dynamiques. Activités avec la TI-n**spire** pour la classe de Première et complements de Seconde*. Paris: Hachette Education.

Aldon, G. (2011). *Mathématiques dynamiques. Activités avec TI-n**spire** CX pour la classe de Terminale*. Paris: Hachette Education.

Artigue, M. (2002). Learning mathematics in a CAS environment: the genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematics Learning*, 7, 245-274.

Artigue, M. (2007). Tecnología y enseñanza de las matemáticas : desarrollo y aportaciones de la aproximación instrumental. In E. Mancera, C. Pérez, *Historia y Prospectiva de la Educación Matemática, Memorias de la XII Conferencia Interamericana de Educación Matemática*, pp. 9-21. Mexico : Edebé Ediciones Internacionales. Reproduced in Cuadernos de Investigación y Formación en Educación Matemática, 8, 13-33 (2011).

Artigue, M. (Ed.) (2009a). Connecting Approaches to Technology Enhanced Learning in Mathematics: The TELMA Experience. *International Journal of Computers for Mathematical Learning*, 14(3).

Artigue, M. (Coord.) (2009b). *Integrative Theoretical Framework – Version C.* Deliverable 18, ReMath Project. http://remath.cti.gr

Artigue M. (2010). ). The Future of Teaching and Learning Mathematics with Digital Technologies. In C. Hoyles, J.B. Lagrange (Eds.), *Mathematics Education and Technology – Rethinking the Terrain. The 17th ICMI Study* (pp. 463-476). New York: Springer.

Artigue, M. (2011). L'impact curriculaire des technologies sur l'éducation mathématique. In A. Ruiz & E. Mancera (Eds.), *Actas del CIAEM XIII*. Recife: Universidade Federale de Pernambuco.

Arzarello, F. (2006). Semiosis as a multimodal process. *RELIME*, 9, 267-299.

Bartolini Bussi M.G., & Mariotti, M.A. (2002). Semiotic mediations in the mathematics classroom: artifacts and signs after a Vygotskian perspective. In L. English, M. Bartolini Bussi, G. Jones, R. Lesh, & D. Tirosh (Eds.), *Handbook of Inernational Research in Mathematics Education, second revised edition.* Mawhah, N.J.: Lawrence Erlbaum.

Borba, M., & Villarreal, M. (2005). *Humans-with-Media and the Reorganization of Mathematical Thinking.* New York: Springer.

Brousseau, G. (1997). *Theory of didactical situations in mathematics.* Dordrecht: Klumwer Academic Publishers.

Buchberger, B. (1990). Should students learn integration rules? *SIGSAM Bulletin*, 24(1), 10-17.

Cedillo, T., & Kieran, C. (2003). Initiating students into algebrawoth symbol-manipulating calculators. In J.T. Fey, A. Cuoco, C. Kieran, L. McMullin &R.M. Zbiek (Eds.), *Computer algebra systems in secondary school mathematics education* (pp. 219-239). Reston, VA: National Council of Teachers of Mathematics.

Chevallard, Y. (1992). Concepts fondamentaux de la didactique : perspectives apportées par une approche anthropologique. *Recherches en Didactique des Mathématiques*, 12(1), 77-11.

Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en Didactique des Mathématiques*, 19(2), 221-266.

Chevallard, Y. (2007). La Dialectique des medias et des milieux. In G. Gueudet & Y. Matheron (Eds.), *Actes du séminaire national de didactique des mathématiques, année 2007* (pp. 344-366). Paris : IREM Paris 7. http://yves.chevallard.free.fr/spip/spip/article.php3?id_article=147

Churchhouse, R.F. (Ed.) (1986). *The influence of Computers and Informatics on Mathematics and its Teaching. ICMI Study Series*. Cambridge: Cambridge University Press.

Cornu, B., & Ralston, A. (Eds) (1992). *The influence of computers and informatics on mathematics and its teaching*. Science and Technology Education. Document Series 44. Paris: UNESCO.

Drijvers, P., Kieran, C., & Mariotti, M.A. (2010). Integrating Technology into Mathematics Education : Theoretical Perspectives. In C. Hoyles and J.B. Lagrange (Eds.), *Mathematics Education and Technology – Rethinking the Terrain. The 17th ICMI Study* (pp. 89-132). New York: Springer.

Drijvers, P. (2011). From 'work-and-walk-by' to 'sherpa-at-work'. *Mathematics Teaching*, 222, 22-26.

Falcade, R., Laborde, C., & Mariotti, M.A. (2007). Approaching functions: The Trace tool as an instrument of semiotic mediation. *Educational Studies in Mathematics*, 66(3), 317-333.

Grugeon-Allys, B., Pilet, J., Chenevotot-Quentin, F., & Delozanne, E. (2012). Diagnostic et parcours différenciés d'enseignement en algèbre élémentaire. *Recherches en Didactique des Mathématiques*, Hors série, 137-162.

Gueudet, G., & Trouche, L. (2009) Towards new documentation systems for teachers? *Educational Studies in Mathematics* 71(3), 199-218.

Gueudet, G., Pepin, B., & Trouche, L. (2012). *From text to 'Lived ressources': Mathematics Curriculum Material and Teacher Development*. New York: Springer

Healy, L., Pozzi, S., & Hoyles, C. (1995). Making sense of groups, computers, and mathematics. *Cognition and Instruction*, 13(4), 505-523.

Hoyles, C., Lagrange, J.B., Le Hung Son, & Sinclair, N. (Eds.) (2006). *Proceedings of the Seventeenth Study Conference of the International Commission on Mathematical Instruction*. Hanoi Institute of Education and University Paris 7.

Hoyles, C., & Lagrange J.B. (Eds.) (2010). *Mathematics Education and Technology – Rethinking the Terrain. The 17th ICMI Study*. New York : Springer.

Lagrange, J.B. (Ed.) (2013). *Les Technologies numériques pour l'enseignement. Usages, dispositifs et génèses.* Toulouse: Editions OCTARES.

Lave, J., & Wenger, E. (1991). *Situated Learning: Legitimate Peripherical Participation*. Cambridge: Cambridge University Press.

Noss, R., & Hoyles, C. (1996). *Windows on Mathematical Meanings: Learning Cultures and Computers*. Dordrecht: Kluwer Academic Publishers.

Pea, R. (1987). Cognitive Technologies for mathematics education. In A.H. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education*, (pp. 89-122). Hillsdale: Lawrence Erlbaum.

Papert, S. (1980). *Mindstorms*. New York: Basic Books.

Rabardel P. (1995). *L'homme et les outils contemporains.* Paris : A. Colin. English version (2002), accessible at http://ergoserv.psy.univ-Paris8.fr

Rabardel, P., & Pastré, P. (Eds.) (2005). *Modèles du sujet pour la conception. Dialectiques activités développement.* Toulouse: Editions OCTARES;

Rasmussen, C., & Nermirovsky, R. (2003). Becoming friends with acceleration: The role of tools and bodily activity in mathematical learning. In N.A. Pateman, B.J. Dougherty, & J.T. Zilliox (Eds.), *Proceedings of the 27th Conference of the*

*International Group for the Psychology of Mathematics Education*, Vol. 1 (pp. 127-135). Honolulu: PME.

Robert, A., & Rogalski, J. (2002). Le système complexe et cohérent des pratiques des enseignants de mathématiques: une double approche. *Revue canadienne de l'enseignement des mathématiques, des sciences et des Technologies,* 2(4), 505-528.

Serres, M. (2012). *Petite Poucette*. Paris: Editions Le Pommier.

Sriraman, B., & English, L. (2010). *Theories of Mathematics Education. Seeking New Frontiers.* New York: Springer.

Trouche, L. (2005). Construction et conduite des instruments dans les aprrentissages mathématiques : nécessité des orchestrations. *Recherches en Didactique des Mathématiques*, 25, 91-138.

Vandebrouck F. (Ed.) (2013). *Mathematics Classrooms: Students' Activities and Teachers' Practices* (pp. 229-245). The Netherlands: Sense Publishers.

Vygotsky, L.S. (1978). *Mind in Society, The Develoment of Higher Psychological Processes.* Cambridge: Harvard University Press.